Vibration Control With Linear Actuator Permanent Magnet System using Robust Control*

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Abstract

This paper describes a $H_\infty$ loop shaping technique in a vibration control system. The prototype of the system consists of vibration body, two permanent magnets, two linear actuators, and control devices. The aim of this prototype system is to suppress the vibration by controlling the air gap between the magnet and the vibration body. The concept of the $H_\infty$ loop shaping technique is to reduce the amplitude of the vibration signal in closed loop system when continuous sine wave disturbance was added. The numerical simulation results verify the effectiveness of the developed vibration suppression system.

Key words: Vibration suppression, $H_\infty$ loop shaping, permanent magnet, actuator

1. Introduction

In the process of plating, coating or rolling of steel sheets, vibration in conveyance often becomes a problem, as sheets are very flexible. As a countermeasure, a vibration suppresser with mechanical contacts is not suitable in such a process. Because objects are easily damaged due to their material makeup such as iron plate which has just been rolled, coated, or plated. Therefore a noncontact suppression mechanism is more suitable for controlling the steel sheets. Problems such as deformation, peeling and uniformless products are minimized. Noncontact vibration control methods which use attractive forces of electromagnets have already been proposed in many papers(1)-(4). The principal weakness of these methods is that the control range is very constricted, because the attractive force of the magnet varies in inverse proportion to the square of air gap length. If the vibration amplitude of the object is large, it becomes impossible to control the object using electromagnets.

A vibration control method using permanent magnet and linear actuators has been proposed.(5),(6). The key to the proposed method is the force control mechanism. A linear actuator drives a permanent magnet and varies the air gap between the magnet and the object. The variation in the size of the air gap changes the attractive force. Since the control range is almost the same as the actuator strokes, we can expect the vibration control range to be correspondingly wide. Nevertheless, uncertainties from the disturbance and parameters error have effect to system response. These uncertainties can be considered in the controller design by applying the robust control $H_\infty$ loop shaping technique.

In this paper, we study the performance in such a method. The outline of the vibration suppression system is introduced and the aim of the system is shown. The principle of control mechanism is explained and numerical simulation results are carried out to demonstrate the performance of the control method and its feasibility.
2. Proposed Vibration Suppression Mechanism

A schematic proposed system illustration of a steel sheet plating or coating process is shown in Fig.1. The steel sheet is fed from the right side of the figure and is directed upwards by a roller moving clockwise. While the steel sheet is being fed into the solution bath, plating or coating is carried out. After the plating process is completed in this way, the steel is seasoned or cooled in the vertical feed. In the seasoning process, the steel is especially sensitive to deformation. Consequently vibration control in the seasoning process is very important.

![Illustration of Plating Process](image1)

![Method to controlling vibration (when the vibration body swing to the right)](image2a)

![Method to controlling vibration (when the vibration body swing to the left)](image2b)

The principle operation of the system is control a permanent magnet to slide in the direction that made a vibration body stop to swing. The method is demonstrate as shown in Fig.2.1. and Fig.2.2.
When the vibration body swing to the right of the equilibrium position as shown in Fig.2.1. The permanent magnet was moved to right side by driving of voice coil motor (VCM). So, the attractive force of the left permanent magnet was made the vibration body straight. On the other hand, the direction of permanent magnet was controlled the attractive force of permanent magnet to reduced a swing of vibration body as shown in Fig.2.2.

3. Experimental Device

An experimental system to examine the performance of the proposed vibration control method was devised. This system was modeled in order to analyse by the linear control theory and to synthesis the control system.

3.1 Experimental System

A photograph of an experimental system is shown in Fig.3. The vibration body supported by parallel spring is the controlled object to suppress vibration. The control force is created by a permanent magnets which is located in the left or the right. Two iron plates are installed on the vibration body. The positions of the vibration body and permanent magnets are measured with each sensor. Fig.4. showing the block diagram of an experiment system consists of vibration body, permanent magnet, voice coil motor, sensor, A/D, D/A, software and hardware of vibration body.
3.2 Modeling of System

Modeling of the experimental system is needed in order to confirm stability, calculate the feedback gains, and permit numerical simulations. In the model, the motion of vibration body is assumed to be translational, as it supported by parallel springs. The modeling of system is shown in Fig. 5

The symbols used in the model are: $z_0, z_1$ are the displacements of the vibration body and permanent magnet, $d_0$ is the air gap width when vibration body is centered between the magnets, $c_0$ and $c_1$ are damping coefficients of the vibration body and the permanent magnets, $k_0$ is parallel spring constant, $m_0$ is the equivalent mass of the vibration body, $m_1$ is mass of the permanent magnet together with voice coil motor (VCM), $f_a$ is force of the actuator, $f_m$ is attractive force of the permanent magnets and $f_d$ is a disturbance force.
The attractive force is determined as follows:

\[ f_m = f_w - f_{ml} = \frac{k}{(d_0 - z_0 + z_1)^2} - \frac{k}{(d_0 + z_0 - z_1)^2} \]  

(1)

when \( k \) is the constant of the magnet. The equation for the motion of vibration body is

\[ m_0\ddot{z}_0 + c_0\dot{z}_0 + k_0z_0 = f_m + f_d \]  

(2)

The equations for the motions of the left and right permanent magnets are

\[ m_1\dddot{z}_1 + c_1\ddot{z}_1 + k_1z_1 = -f_m + f_a \]  

(3)

When the inputs of the system are defined by the forces of the actuators, the model is represented by Eq. (1),(2) and (3). The system inputs is the actuator force \( f_a \), the outputs are the displacements of the vibration body and permanent magnets.

A linearized state space model of the system is:

\[ x' = Ax + b_1u_1 + b_2u_2 \]  

(4)

\[ y = Cx \]  

(5)

where \( u_1 = f_a \), \( u_2 = f_d \) are input and state vector \( x = [z_0,\dot{z}_0, z_1,\dot{z}_1]^T \) and linearization of a attractive force is \( \tilde{f}_m = k_mz_0 - k_mz_1 \), \( k_m \) is constant

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-k_m - k_0 & c_0 & -k_m & 0 \\
m_0 & -m_0 & m_0 & 0 \\
0 & 0 & 0 & 1 \\
-k_m & 0 & k_m - k_0 & c_1 \\
m_1 & -m_1 & m_1 & 0
\end{bmatrix}, \quad b_1 = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
-1 \\
0
\end{bmatrix}, \quad b_2 = \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix}
\]

And \( C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix} \)

Parameters used in simulation are aligned to the actual experimental device and these value are \( m_0 = 0.128[kg], m_1 = 0.0763[kg], d_0 = 0.007[m], k_0 = 25.79[N/m], c_0 = 0.03[Ns/m], k_m = 3.67 \times 10^{-7}[Nm^2], k_m = 33.33[N/m] \).
3.3 $H_\infty$ Loop Shaping Design Procedure (LSDP)

In this paper we use the proposed technique of McFarland D.C. and K. Glover (7). According to the standard procedure of $H_\infty$ loop shaping. The following steps can be applied to design the $H_\infty$ loop shaping controller.

Step 1. Loop Shaping: We selecting shaping function $W_1$ (precompensator) and/or $W_2$ (postcompensator). When $W_1$ and $W_2$ are the singular values and $G$ is the nominal plant, The combined of $W_1$, $W_2$ and $G$ is the shaped plants $G_s$, as shown in Fig.6.

![Fig.6. The robust stabilization.](image)

$$G_s = W_2 G W_1$$ (6)

In this paper we selected $W_1 = \frac{1}{0.05s + 1}$, gain is 0.1 and $W_2 = I$

Step 2. Robust Stabilization: Firstly, we calculated the maximum stability margin $\varepsilon_{\text{max}}$ by equation (7)

$$\varepsilon_{\text{max}}^{-1} \triangleq \inf_{\text{stabilizing}} \left\| \begin{bmatrix} K & I \end{bmatrix} (I - G_s K)^{-1} \tilde{M}^{-1}_s \right\|_\infty$$ (7)

When $\tilde{M}_s$, $\tilde{N}_s$ are normalized coprime factor of $G_s$ and $G_s = \tilde{M}_s^{-1} \tilde{N}_s$, $\tilde{M}_s \tilde{M}_s^T + \tilde{N}_s \tilde{N}_s^T = I$ and $\| \cdot \|_\infty$ denoted to the $H_\infty$ norm. to determine $\varepsilon_{\text{max}}$. If $\varepsilon_{\text{max}} \ll 1$ indicates that $W_1$ or $W_2$ designed in step 1 are incompatible with robust stability requirement. If $\varepsilon_{\text{max}}$ is not satisfied we will reselected $W_1$ or $W_2$. Next, we synthesis a stabilizing controller $K_\infty$ which satisfies equation (8)

$$\left\| \begin{bmatrix} K_\infty & I \end{bmatrix} (I - G_s K)^{-1} \tilde{M}^{-1}_s \right\|_\infty \leq \varepsilon^{-1}$$ (8)

In this system when $G$ is vibration control system as shown in equation (1) to equation (5), $G_s$ in equation (6), $W_1$, $W_2$ was selected then we get $\varepsilon_{\text{max}} = 0.19225$
Step 3. \textbf{Fine the final feedback controller:} \( K \): The final controller \( K \) can be obtained by the combination of \( W_1, W_2 \) and \( K_\infty \)

\[
K = W_1 K_\infty W_2
\]  

(9)

In this step we get the controller \( K \) is

\[
K = [K]
\]

\[
= [K_a, K_b, K_c, K_d]
\]

3.4 Comparisons between PD Controller and Robust Controller \( H_\infty, \) LSDP

The consideration at disturbance force \( f_d \) is an input of the system, displacement of a vibration body \( z_0 \) and permanent magnet \( z_1 \) are outputs as shown in Fig.7. where \( f_a \) is attractive force, \( P(s) \) is vibration control system, and \( K(s) \) is feedback gain

\[
f_a = -(k_{p0} \ddot{z}_0 + k_{d0} \dot{z}_0 + k_{p1} \ddot{z}_1 + k_{d1} \dot{z}_1)
\]  

(10)

where \( k_{p0} = 562, k_{p1} = 3252.2, k_{d0} = -126.44 \) and \( k_{d1} = 48.8 \) frequency response of the system when input of the system is disturbance force \( f_d \) and output is displacement of vibration body as shown in Fig.8.
If \( K(s) \) is \( H_{\infty} \), robust control frequency response as shown in Fig. 9.

The numerical simulation in MATLAB with Simulink we simulated in the condition is vibration body has a disturbance force, a force is sinusoidal function with 1N(peak), frequency is 5 Hz. We put the disturbance force on the system at 20 second and system without controller as shown in Fig.10. Next, the system has controller as shown in Fig.11. and Fig.12. are PD controller, Fig.13. and Fig.14. are $H_{\infty}$ robust controller.

![Fig.10](image1)

**Fig.10.** Displacement signal of vibration body and permanent magnet when the system without controller and disturbance force is sinusoidal function 5 Hz, 1 N(p)

![Fig.11](image2)

**Fig.11.** Displacement signal of vibration body when the system with PD controller and disturbance force is sinusoidal function 5 Hz, 1 N(p)

![Fig.12](image3)

**Fig.12.** Displacement signal of permanent magnet when the system with PD controller and disturbance force is sinusoidal function 5 Hz, 1 N(p)
Fig. 13. Displacement signal of vibration body when the system with $H_{\infty}$ robust controller and disturbance force is sinusoidal function 5 Hz, 1 N(p)

Fig. 14. Displacement signal of permanent magnet when the system with $H_{\infty}$ robust controller and disturbance force is sinusoidal function 5 Hz, 1 N(p)

Fig. 11. and Fig. 12. are the displacement of vibration body and permanent magnet when controller is PD control. We can see that the system take a short time to stop vibrated, Fig. 13 and Fig. 14 are the displacement of vibration body and permanent magnet when controller is $H_{\infty}$ robust control. The comparison between PD controller and $H_{\infty}$ robust controller we can see the performance of $H_{\infty}$ robust controller can reduced the disturbance signal more than PD controller.

5. Conclusion

In this paper, we proposed a $H_{\infty}$ robust control using LSDP to design controller to control the vibration system. First, the vibration control system has been modeled and linearized. After that the $H_{\infty}$ robust control was designed and satisfied by robust closed-loop stability. Finally, we can see the numerical simulations has been proven controller can suppress vibration.
References

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