Feasibility Analysis of Two Iron Balls’ Simultaneous Suspension Using Flux Path Control Mechanism*

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Abstract
This paper proposes a noncontact magnetic suspension system for levitating two different weight iron balls simultaneously using a flux path control mechanism. In this system, the suspension force is generated by a disk-type permanent magnet, and controlled by varying the angle of the permanent magnet that is driven by a rotary actuator. In this paper, first, the suspension principle is explained, and the prototype is introduced. Second, the characteristics of this system are examined by some basic experimental results. Third, a model is created, and the controllability is proved, theoretically. Finally, the numerical simulation results of suspension are shown and discussed.

Key words: Magnetic Suspension, Noncontact, Permanent Magnet, Flux Path Control, Actuator

1. Introduction
Magnetic suspension is technology for supporting or manipulating objects without contact by means of magnetic forces. Magnetic suspension system has many advantages, which are no contact, no friction, no dirt and lubrication free. Using these advantages, many magnetic suspension mechanisms have been proposed (1). Moreover, these mechanisms have been developed and used in noncontact conveyance vehicles, high-speed transportations and motors, dirt-free factories and outer space. These magnetic levitation systems use various methods to control the suspension force. Two types of systems are electromagnetic suspension systems, which control the coil current so as to change the magnetic force in order to levitate an object stably; and mechanical magnetic suspension systems, which use permanent magnets to control the magnetic reluctance so as to vary the suspension force in order to achieve stable suspension. Moreover, there are two methods for controlling magnetic reluctance in mechanical magnetic suspension systems (2), i.e. changing the air gap between magnets and ferromagnetic objects by moving the permanent magnet (3), and varying the flux by changing the flux path of the magnetic circuit. Many researchers have proposed magnetic suspension systems using flux path control method. Mizuno et al. have proposed a magnetic suspension system with a permanent magnet and three flux-path control modules consisting of a ferromagnetic plate, a voice coil motor (VCM), and a
displacement sensor (4). This system controls the attractive force of the magnet by changing the flux path with the flux-path control modules. Ueno and Higuchi presented a magnetic levitation technique using flux path control method with a composite of magnetostrictive and piezoelectric materials (5).

Moreover, some multiple magnetic suspension systems have been proposed. Yamamoto et al. have discussed a series magnetic suspension system with an electromagnet, a permanent magnets and a magnetic target (6). Yamamoto’s suspension system realized an indirect suspension of a magnetic target through an actively controlled permanent magnet, whose position is controlled by the electromagnet. Sakurada et al. have proposed a parallel magnetic suspension of two-floater using one signal amplifier (7). Moreover, Sakurada has confirmed that there were minus feedback gains in the current control of the parallel two-fold magnetic suspension.

This paper proposes a noncontact magnetic suspension system for levitating two different weight iron balls simultaneously using flux path control method (8)(9). In this system, the magnetic flux is generated from a disk-type permanent magnet, and passes through two iron cores to arrive two iron balls. Therefore, the suspension force of two cores can be changed simultaneously at same variation rate by varying the angle of the permanent magnet that is driven by a rotary actuator. This proposed suspension system of two-ball is similar to the parallel-type double inverted pendulum system, which consists of two different length pendulums and a cart or a linear motor (10). In the double inverted pendulum system, two pendulums can be maintained the balance at the up-right position through the motion control of the cart in one degree.

In this paper, first, the suspension principle of two iron balls is explained, and a prototype is introduced. Second, the characteristics of this system are examined by some basic experimental results. Third, a model is created, and the controllability is proved, theoretically. Finally, the simulation results of suspension are shown and discussed.

2. Suspension Principle

The suspension principle of this magnetic suspension system can be understood from Fig. 1, a schematic diagram showing a disk permanent magnet, two iron cores and two iron balls. The permanent magnet is in the center of diagram, and has an N pole of half part and an S pole of opposite half part in radial direction. Two iron cores, resembling a pair of opposite letters of “F”, and are installed beside the permanent magnet. Two iron balls have different size, and are located under the two cores, respectively.

First, the followings will be assumed:

1. The distance between two iron balls is far enough, and comparing with the attractive force from the iron cores, the attractive force between two balls can be neglected.

2. The rotation angle of permanent magnet is 0 degree at the situation shown in Fig.1 (a).

Fig. 1 (a) shows that the magnetic poles of the permanent magnet are aligned in the vertical direction, and the N pole is at the upper side and the S pole is in the lower side. In this case, the facing angle of the N pole and S pole to each core are same, so all magnetic flux comes from the N pole and is absorbed into the S pole through the upper part of each core, respectively. There is no magnetic flux arrive at two iron balls, so zero attractive force generates between the cores and the levitated iron balls. However, Fig. 1 (b) shows the permanent magnet rotated 20 degree, the facing angle of the N pole becomes bigger than the S pole in the right core, and that is reverse in the left core. Since that, some flux coming from N pole passes the right core, and arrives at the right ball; and some flux passes through the left iron ball and left core, and returns to the S pole. Consequently, the attractive force is generated between cores and iron balls. Moreover, Fig. 1 (c) shows the permanent magnet
rotated 40 degree. The flux flowing through two iron balls becomes more as the rotation angle of magnet becomes larger. As a result, the attractive force between two cores and two iron balls can be changed by means of varying the rotation angle of permanent magnet simultaneously.

In addition, since the natural frequency of an iron ball depends on its weight, and the small ball has a higher natural frequency. Since the rotation of the magnet affects the two iron balls directly, if the magnet rotates same angle, the small ball tends to respond intensively and moves a large distance.

Consequently, simultaneous suspension of two iron balls can be realized using the proposed flux path control method, theoretically.

3. Experimental Prototype

An experimental prototype of the proposed magnetic suspension system was constructed, and the photograph of the prototype is shown in Fig. 2. This prototype consists mainly of a disk permanent magnet, a rotary actuator containing a gear reducer and an encoder, a pair of opposite F-type permalloy cores, two different size iron balls and two eddy current sensors. The disk permanent magnet, which is in the center of the opposite F-type cores, is a neodymium magnet and magnetized in the radial direction. The diameter of the magnet is 30 mm and the thickness is 10 mm. The magnet is installed on the shaft of a rotary actuator. The rotary motor is at behind of the magnet, cannot be seen in Fig. 2. The encoder of the actuator measures the rotation angle of the magnet. The thickness of the two permalloy cores is 10 mm that is same as the magnet. The diameter of iron ball under the left core is 20 mm, and the weight is 35.8 g. The diameter of iron ball under the right core is 30 mm, and the weight is 110 g. The position of the two levitated balls is measured by two eddy current sensors, of which the measurement range is 3.5 mm and the measurement error is less than 0.007 mm.

The configuration of the system is shown in Fig.3. The position of two balls and the
angle of magnet are measured, and returned to a DSP controller. Then the DSP controller calculates the current to control the rotary actuator. The actuator drives the magnet rotate to an appropriate angle. As a result, two iron balls will be suspended without contact.

4. Basic Experiment

For examining the characteristics of the experimental prototype of this proposed magnetic suspension system, the magnetic flux density of the permanent magnet, the magnetic flux density near the surface of iron ball and attractive force between cores and iron balls were measured.

4.1 Magnetic Flux Density of Permanent Magnet

To examine the magnetic characteristic of the permanent magnet and the influence of the distance from the magnet, the magnetic flux density of the permanent magnet was measured using a gauss-meter in the radial direction of the disk magnet. The flux density was recorded, while the magnet rotated in one revolution and increased at 10 degrees as a step, and the distance from the radial surface of the magnet was changed from 1 mm to 8 mm and increased at 1 mm as a step. The measurement results are shown in Fig. 4. From the results, it can be seen that the flux density curves resemble sine curves at all points, and smaller distance yields greater flux density. It means that the distance between magnet and iron cores is smaller; the attractive force generated by cores will be larger. In this prototype, the distance between the magnet and cores was set in 2 mm for considering the radial precision of magnet rotation.
4.2 Magnetic Flux Density between Cores and Iron Balls

The magnetic flux density was examined using a gauss-meter between the core and iron ball at the point 0.5 mm from the surface of iron ball. When the magnet is rotating, the distance between the core and the iron ball is changing. In the experiment, the step of the magnet’s angle is 10 degrees and the range is one revolution. The step of the distance is 0.5 mm and the measured range is from 2 mm to 10 mm. The used iron balls were same as the ones were introduced in experimental prototype (as shown in Fig. 2). When measuring the flux density of one side, the other ball was installed in the other side, and the distance was 2 mm. The results of two iron balls are shown in Fig. 5. In the figure, the horizontal axis expresses the rotation angle of permanent magnet, the vertical axis expresses the magnetic flux density on the surface of iron ball, and the parameters on the top of figures express the distance between the core and iron ball. The graph of measured flux density resembles sine curve according to the magnet’s angles at all distance, and smaller distance yielded greater flux density. The flux density on the surface of big iron ball is always large. Moreover, the direction of the flux is changing at about 180 degrees.

4.3 Attractive Force between Cores and Iron Balls

In addition, the relationship between attractive force, the rotated angle of the magnet and the distance was examined. In this experiment, the two iron balls were installed on two force sensors, and the measurement range was from 1 mm to 8 mm. Others condition was same as the experiment of flux density measurement. The results of left ball and right ball are shown in Figs. 6 and 7, respectively. As shown in the figure, the force curve in each
position varies according to the rotated angle of the magnet, and has 2 maximum points at about 90 degrees and 270 degrees and has 2 approximately zero points at about 0 degree and 180 degrees. The forces become smaller when the gap becomes larger, and the attractive force of big ball is large.

All results indicate that, varying the magnet’s angle can control the attractive force between the cores and the levitated object, and the generated attractive force at same angle of magnet relates with the size of iron ball. The flux arrives the big iron ball much, and generates a large suspension force.

5. Theoretical Feasibility Analysis

In order to investigate the feasibility of the proposed magnetic suspension system, a model is setup, and the controllability of the experimental system are checked.

5.1 Suspension System Modeling

For setting up the model, first, the followings are assumed:

1. One half of the permanent magnet is N pole and the other half part is S pole.
2. The flux coming out from the magnet and absorbed to the magnet is in proportion to the area facing to the iron core.
3. The magneto-resistance of the iron cores is small enough and all magnetic flux passes through the two iron cores.

Second, according to the result in Fig.5, the relationship of the flux flowing through
iron ball, the rotation angle of permanent magnet and the distance between core and iron ball can be considered as:

\[ Q = k_0 \sin \theta \frac{\sin \theta}{d} \]  

(1)

where,

- \( Q \) : magnetic flux flowing through the levitated object
- \( k_0 \) : constant of magnetic flux
- \( \theta \) : rotated angle of the magnet (positive is the clockwise direction)
- \( d \) : distance between core and iron ball

Moreover, according to the results in Fig.6 and Fig.7, we can consider that the attractive force is in proportion to the square of the flux. Then the attractive force between core and iron ball can be expressed as,

\[ f_a = k_a \sin^2 \theta \frac{\sin^2 \theta}{d^2} \]  

(2)

where,

- \( f_a \) : attractive force between core and iron ball
- \( k_a \) : constant of attractive force

Third, a model of this suspension system is shown in Fig.8. According to the model and the assumptions, the motion equations of the magnet and two iron balls are, respectively,

\[ m_1 \ddot{z}_1 = f_{a1} - m_1 g \]  

(3)

\[ m_2 \ddot{z}_2 = f_{a2} - m_2 g \]  

(4)

\[ J \ddot{\theta} = c_1 \dot{\theta} + f_\theta \]  

(5)

\[ f_{a1} = k_{a1} \frac{\sin^2 \theta}{(d_1 - z_1)^2} \]  

(6)

\[ f_{a2} = k_{a2} \frac{\sin^2 \theta}{(d_2 - z_2)^2} \]  

(7)

\[ f_a = k f \]  

(8)
where,

- $m_1$: mass of left iron ball
- $m_2$: mass of right iron ball
- $z_1$: displacement of left iron ball
- $z_2$: displacement of right iron ball
- $f_{jm}$: attractive force between the left core and iron ball
- $f_{jn}$: attractive force between the right core and iron ball
- $k_{ai}$: constant of the attractive force between the left core and iron ball
- $k_{aj}$: constant of the attractive force between the right core and iron ball
- $d_1$: distance between the left core and iron ball
- $d_2$: distance between the right core and iron ball
- $J$: moment of inertia of motor and magnet
- $c_i$: damping constant of motor and reducer.
- $f_a$: driving force of motor
- $k_t$: torque constant of motor
- $i$: input current of motor

### 5.2 Analysis of Controllability

In the motion equations of this system, the attractive force $f_m$ is represented as a nonlinear function of the rotation angle of permanent magnet and the distance between core and iron ball. Moreover, the attractive force becomes larger as the rotation angle increases and becomes smaller as the distance decreases. Through linearization of the attractive force around the equilibrium position, the linear relationship of attractive force, rotation angle and the distance can be obtained.

$$\Delta f_{jm} = \frac{2k_{jm} \sin(\theta_0)}{d_{j0}} \Delta z_j + \frac{k_{jm} \sin(2\theta_0)}{d_{j0}^3} \Delta \theta$$

where,

- $j = 1, 2$
- $\Delta f_{jm}, \Delta z_j, \Delta \theta$: small value around the equilibrium position
- $\theta_0, d_{j0}$: value at the equilibrium position

According to the equations from Eq. (3) to Eq. (9), the state space equation and the output equation are represented as:

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

where

$$x = \begin{pmatrix} z_1 & \dot{z}_1 & z_2 & \dot{z}_2 & \theta & \dot{\theta} \end{pmatrix}^T$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{2k_{ai} \sin(\theta_0)}{m_1 d_1^3} & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{k_{ai} \sin(2\theta_0)}{m_1 d_1^3} & 0 \\
0 & 0 & \frac{2k_{aj} \sin(\theta_0)}{m_2 d_2^3} & 0 & \frac{k_{aj} \sin(2\theta_0)}{m_2 d_2^3} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & \frac{c_i}{J} \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{k_t}{J} \end{pmatrix}^T$$
\[ C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \]

\[ u = i \]

The controllability matrix of the system of Eq. (10) is

\[ P_c = \begin{pmatrix} B & AB & A'B & A'B & A'B \end{pmatrix} \]

Then determinant of the controllability matrix is

\[ \text{det}(P_c) = -\frac{4g^6k_i^6 \sin^4(2\theta_i)(d_i - d_j)^2}{d_1^2d_2^2J^6\sin^3\theta_i} \]

Therefore, when \(d_i\) and \(d_j\) are different in the suspension state, the determinant of the controllability matrix is nonzero. According to the characteristic of this suspension system, many conditions can cause \(d_i\) and \(d_j\) different, such as the different magneto resistance material or size of two iron cores, the different material and size of two iron balls. However, this paper just investigates the condition of two iron balls with same material and different size. Consequently, the proposed magnetic suspension system is controllable, and suspension feasibility is verified, theoretically.

5.3 Control System

The control system of the suspension system is shown in Fig. 9. According to feedback of the positions of two iron balls and the rotation angle of magnet, this control system uses three PD controllers to calculate the current of the rotary motor.

6. Numerical Simulation

In this feasibility study, first, the feedback gains were calculated using the LQR (linear quadratic regulator) full state feedback control law with the linear mathematical model of Eq. (10). Second, the numerical simulation was carried out with a nonlinear attractive force.

6.1 Calculation of Feedback Gains

All the parameters using in the numerical simulation are shown in Table 1, which are
measured from the experimental prototype. Moreover, the values of $k_{1m}$ and $k_{2m}$ are measured from Figs. 6 and 7.

Many methods can design the controller for a linear system. In terms of linear control theory, due to this proposed system is controllable and observable, the LQR can be used to get the feedback gains.

Based on the characteristics of the system and Eq. (10), we chose the state weighting matrix $Q$ and input weighting matrix $R$ as following:

$$Q = \begin{bmatrix} 100000000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1000000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(13)

$$R = 1$$

(14)

Using MATLAB software, the feedback gain $K$ is calculated as following:

$$K = \begin{bmatrix} -1827200 \\ -30551 \\ 2447300 \\ 34717 \\ 364.63 \\ 0.82 \end{bmatrix}$$

(15)

### 6.2 Numerical Simulation

Using the nonlinear attractive force model and the calculated feedback gain, the simulation was carried out. The simulation results show in Figs. 10 and 11. In the simulation, when the two iron balls were levitated, a 0.01 mm step value was added to the two iron balls as a disturbance, respectively. After the step was added, the input current of motor, the rotational angle of magnet, and two iron balls’ displacements were recorded. The step was added at 0.05 second, and the responses were recorded until 0.3 second. The varieties of current, angle, displacements, and the disturbances were plotted, with the upward direction as positive.

Fig. 10 shows the simulation results when the step is applied to the left iron ball. In the figure, after applying the step, the system reinstates the steady suspensions of two iron balls quickly. The step drives the left iron ball move upward, and for maintaining the suspension force equal with gravitational force of the iron ball, the rotational angle of magnet becomes small a little. However, the variation of rotational angle also directly affects the suspension of the right iron ball. In order to also suspend the right iron ball stably, the system moves the right iron ball up a little distance at same time with the left iron ball. Since the gravitational force of iron balls is supported by the two iron cores fixed on the frame, the
motor just drives the magnet's rotation, and the input current of motor returns to zero after returning to the equilibrium state.

Fig. 11 shows the simulation results when the step is applied to the right iron ball. The results in Fig. 11 indicate that, the suspensions of two iron balls can also be realized stably after applying the disturbance to the right iron ball. However, the movements of two iron balls are downward, and the final angle of the magnet increases, which are reverse to the results in Fig. 10. These differences are caused by the different signs of the feedback gains as shown in Eq. (15). In Eq. (15), the first two feedback gains are minus for controlling the left ball; the middle two feedback gains are plus for controlling the right ball; the last two
feedback gains are plus for controlling the disk magnet.

Consequently, all the simulation results indicated that this proposed system could simultaneously suspend two iron balls steadily.

7. Conclusion

A simultaneous suspension system of two iron balls was proposed using a disk type permanent magnet and a rotary motor. This system controlled the suspension force by varying the flux path. An experimental prototype was introduced, and some basic experiments were carried out. According to the results of the basic experiments, a mathematical model was created, and the feasibility of the proposed system was verified by the theoretical examinations. A numerical simulation was completed, and the simulation results indicated that the simultaneous suspension of two iron balls could be realized using this proposed suspension system, theoretically.

References