Vibration Control With Linear Actuator Permanent Magnet System using LQR Method*

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Abstract
This paper proposes a vibration suppression mechanism consisting of permanent magnets, actuators, sensors and a controller. The aim of this proposed system is to reduce vibration and/or deformation of thin steel sheets by controlling the air-gap between the permanent magnets and the steel sheets. The feedback control of the system was designed by means of the LQR method. In this study, the proposed vibration suppression mechanism was designed, the prototype was constructed for experimental confirmations, feasibility of the model of prototype and the design of the controller was analyzed, and numerical simulations and experimental examinations were carried out to verify the effectiveness of the controller designed by LQR method. The simulations and experiments were performed under initial disturbance conditions with PD control and without PD control. All results verified that the system effectively suppressed vibration.

Key words: Vibration Control, Permanent Magnet, Linear Voice Coil Motor, Linear Quadratic Regulator, Free Vibration

1. Introduction

In the process of plating, coating or rolling of steel sheets, vibration in conveyances often leads to problems of deformation, peeling and non-uniform products, because the sheets are very flexible. Due to this, a vibration suppressor with mechanical contacts is not suitable as a countermeasure. Objects are easily damaged due to their material makeup when they have just been rolled, coated or plated. Therefore, a noncontact suppression mechanism is more suitable for controlling the minimization of deformation of the steel sheets and can minimize such problems. Noncontact vibration control methods which use attractive force of electromagnets have been already proposed (1)-(4). The principal weakness of these methods is that the control range is very constricted, because the attractive force of the magnet varies in inverse proportion to the square of air-gap length. If the vibration amplitude of the object is large, minimizing deformation by means of electromagnets is ineffective for the control of the object. On the other hand, a control method using permanent magnets located at the desired iron sheet passage has also been proposed (5). However, this method uses passive control and can not be successfully suppressed under large disturbances.

A noncontact suspension system using permanent magnets and linear actuators has been proposed (6). The key element of the design is the force control mechanism. A linear actuator drives a permanent magnet and varies the air-gap between the magnet and the object. Variation of the size of the air-gap changes the attractive force. Here the vibration
control system uses this type of force control mechanism for vibration suppression, since the control range is almost the same as of the actuator stroke length. In this system vibration control range is expected to be correspondingly wide.

In this paper, a new type of vibration suppression mechanism was proposed and the performance of the vibration control systems was studied. The structure of the vibration suppression system is introduced and the aim of the system is shown. The principle of control mechanism is explained and numerical simulations and experimental examinations are carried out to demonstrate the successful performance and feasibility of the control method.

2. Proposed Vibration Suppression Mechanism

A schematic diagram of a vibration suppression mechanism for plating or coating process is shown in Fig.1. The steel sheet is fed from the right side of the figure and is directed upwards by a roller. While the steel sheet is being fed into the solution bath, plating or coating is carried out. After the plating process is completed, the steel is seasoned or cooled in a vertical feed. During the seasoning process, the steel is especially sensitive to deformation. Consequently vibration control in the seasoning process is very important.

![Fig. 1 Illustration of plating process](image)

In this paper, a new type of vibration suppression mechanism, as shown in Fig. 1 is proposed. The mechanism consists of two permanent magnets, two linear actuators, sensors which measure the steel sheet displacement, and a controller. The sensors and controller are omitted in the figure. The intention of the system is to reduce vibration caused by the roller feed mechanism, air wipers, and high speed plating. Air wipers, represented by the green element in the figure, adjust the thickness of the plating.

The design of the prototype of vibration suppression mechanism is illustrated in Fig. 2. In this figure, the vertical sheet in the center represents the steel sheet and is labeled “Vibration body”. The magnets on both sides are driven in the horizontal direction by actuators, which are labeled “VCM”. The principle of the vibration suppression method is as follows. When the vibration body swings to the right of the equilibrium position, as shown in Fig.2 (a1), the left actuator (VCM) drives the left magnet to the right, the displacement between the left magnet and vibration body is reduced, and the left attractive force becomes larger. Consequently, the magnetic force caused the vibration body return to the original equilibrium position. Similarly, when the vibration body swings to the left, as shown in Fig. 2 (b1), the right actuator drives the right magnet to the left, the attractive force on the right side of the vibration body becomes larger and this force caused the object to return to the equilibrium position. The attractive force of the permanent magnet can suppress vibration and deformation of the object.
3. Experimental Device

An experimental system to examine the performance of the proposed vibration control mechanism was manufactured. The photograph of the system is shown in Fig. 3. The system consists of a vibration body, two permanent magnets installed with a slider, a voice coil motor (VCM) and sensors. The vibration body is structured as a parallel spring made by phosphor bronze and installs steel plates on the both sides facing to the permanent magnets. Two permanent magnets are installed on the slider driven by the VCM. The system is a simultaneous drive system that is a little different from the principal system shown in Fig. 1. This prototype is, however, the first step for the proposed system. There is no problem for verifying the feasibility. Two sensors are installed for measuring the displacements of the vibration body and the slider. A laser sensor is used for the motion of the vibration body and an eddy current sensor is used for the motion of magnets which is the same as the VCM movement.

The control system block diagram is shown in Fig. 4. As seen in the figure, the controller is using a DSP. It calculates the generating force of the VCM from two sensor signals and the output is current signal for the amplifier. The amplifier changes the signal to the current for the VCM and the VCM changes the currents to the force proportionally. For DSP programming, MATLAB with SIMULINK and Pass/C67 were used.

This system was modeled in order to analyze the system, to adopt the linear control theory, and to synthesize the control system. For modeling, the specifications should be figured out. The relationship between attractive force and air-gap of a permanent magnet and a steel plate on the vibration body is shown in Fig. 5. As shown in the figure, the relationship is nonlinear. As magnets are installed at both sides of the vibration body in the actual prototype system, the generating force is a resultant of two magnets as shown in Fig. 6. Figure 6(a) shows the resultant when the air-gap of the equilibrium position is 12.5 mm. and Fig. 6(b) shows the resultant of 20 mm. air-gap. Other specification values of the prototype are indicated in Table 1.
Fig. 3 Photograph of an experimental prototype system

![Prototype System](image)

Fig. 4 Block-diagram of a system

![Block-diagram](image)

Fig. 5 Relationship between attractive force and displacement of permanent magnet

![Force-Displacement Graph](image)
Fig. 6 The test results of the attractive force of permanent magnets

Table 1 Parameters of the vibration control system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_v$ : damping coefficient of the vibration body</td>
<td>0.02 Ns/m</td>
</tr>
<tr>
<td>$c_p$ : damping coefficient of the permanent magnet</td>
<td>0 Ns/m</td>
</tr>
<tr>
<td>$k_v$ : constant of vibration body</td>
<td>150 N/m</td>
</tr>
<tr>
<td>$k_p$ : constant of permanent magnet</td>
<td>0 N/m</td>
</tr>
<tr>
<td>$m_v$ : mass of vibration body</td>
<td>0.047 kg</td>
</tr>
<tr>
<td>$m_p$ : mass of permanent magnet</td>
<td>0.292 kg</td>
</tr>
<tr>
<td>$k_{m1}$ : constant of linearization (equilibrium position=12.5 mm)</td>
<td>86.66 N/m</td>
</tr>
<tr>
<td>$k_{m2}$ : constant of linearization (equilibrium position=20 mm)</td>
<td>33.85 N/m</td>
</tr>
</tbody>
</table>
4. Systems modeling

In the model, the motion of the vibration body, as supported by a parallel spring is assumed to be translational. An illustration of the model is shown in Fig. 7. The symbols used in the model are:

- $z_v, z_p$: displacement of the vibration body and permanent magnet,
- $d_0$: air-gap length when the vibration body is centered between the magnets,
- $f_m$: resultant force of magnets,
- $f_{vcm}, f_d$: actuator generating force and the disturbance force.

The resultant force is assumed as inverse proportion to the square of gap length.

$$f_m = \frac{k}{(d_0 - z_v + z_p)^2} - \frac{k}{(d_0 + z_v - z_p)^2}. \quad (1)$$

The motion equation of the vibration body is

$$m_v z''_v = -k_v z_v - c_v z'_v + f_m + f_d. \quad (2)$$

The motion equation of the slider is

$$m_p z''_p = -k_p z_p - c_p z'_p + f_{vcm} - f_m. \quad (3)$$

![System modeling diagram](image_url)

Fig. 7 System modeling of a system

The inputs of the system are defined as the forces of the actuator (VCM) $f_{vcm}$ and the disturbance $f_d$. The outputs are the displacement of the vibration body $z_v$ and the slider $z_p$. The model can be represented by Eqs. (1), (2) and (3).

By linearization of the attractive force of the magnets, the resultant force can be represented by

$$f_m = k_m (z_v - z_p). \quad (4)$$

Equations (2) and (3) become as

$$m_v z''_v = -k_v z_v - c_v z'_v + k_m (z_v - z_p) + f_d, \quad (5)$$

$$m_p z''_p = -k_p z_p - c_p z'_p + f_{vcm} - k_m (z_v - z_p). \quad (6)$$

The system can be considered as a block diagram shown in Fig. 8. There are PD controllers in the loops of vibration body and the magnets. The feedback gains are calculated by mean of the LQR control theory. Using the LQR method, a state space model
can be derived from Eqs. (4) – (6) as;

\[ x' = Ax + Bu, \]
\[ y = Cx. \]  

Where, \( u = f_{vcm} \), \( y = [z_v, z_p]^{T} \), state vector \( x = [z_v, z_p, z'_v, z'_p]^{T} \).

**Fig. 8 Block-diagram of a vibration control system**

5. Numerical Simulations

Numerical simulations were carried out with nonlinear attractive force. The results of the performances of vibration suppression were recorded when the initial displacement of the vibration body is set at 3 mm. The conditions of the simulations were; 1) without permanent magnets 2) with permanent magnet \( (d_0=20 \, \text{mm}) \) and without feedback control, 3) with permanent magnet \( (d_0=20 \, \text{mm}) \) and with feedback control, and 4) with permanent magnet \( (d_0=12.5 \, \text{mm}) \) and with feedback control. In case of condition 3) and 4), the weighting matrices for calculating the optimal gains were equal.

**Fig. 9 Displacement signal of vibration body, the conditions are without feedback control and without permanent magnet**
Fig. 10 Displacement signal of vibration body and permanent magnet, the conditions are with permanent magnet, without feedback control and the air-gap of the equilibrium position \( d_0 \) is 20 mm

Fig. 11 Displacement signal of vibration body and permanent magnet, the conditions are with feedback control and the air-gap of the equilibrium position \( d_0 \) is 20 mm
According to the simulation results in the case without feedback control as shown in Figs. 9 and 10, the red line in Fig. 9 is a displacement of vibration body, the vibration body was vibrated at natural frequency and stopped vibrating more than 8 seconds. Figure 10 shows the displacement of vibration body and the permanent magnet. It should be noted that the amplitude and natural frequency graphs are different from Fig. 9. This can be explained by the fact that the attractive force of permanent magnet and the air-gaps between vibration body and permanent magnets in this figure are 20 mm.

The simulation results in Figs. 11 and 12 show the performance of feedback control. In Fig. 10, the equilibrium position of the air-gap is 20 mm. The attractive forces of permanent magnet have changed in order to make the vibration body return to the equilibrium position. Thereafter, the vibration body reached the equilibrium position again at 2 seconds. The lower plot shows the transient response of the simulation result. In addition, the simulation condition was varied of the equilibrium position ($d_0$) to 12.5 mm. as shown in Fig. 12. In this figure, the air-gap was decreased and the attractive force of the permanent magnet was increased, so the systems stopped vibrating sooner. The lower plot shown the transient response of the simulation result, as can be observed in Fig. 12 that the vibration body stopped vibrating at 1 second.

All of simulation results proved the success of the design of feedback control. Because the air-gap between permanent magnets and vibration body in Fig. 11 is larger than in Fig. 12, the vibration body in Fig. 12 stopped vibrating sooner than in Fig. 11. However, the vibration body in the system with feedback control in Figs. 11 and 12 will stopped vibrating sooner than the one without feedback control.
6. Experimental Results

Vibration control experiments were also conducted using the experimental prototype shown in Fig. 3. Trials under various conditions and equivalent to the numerical simulations were completed.

![Displacement signal of vibration body, the system conditions are without feedback control and without permanent magnets](image)

**Fig. 13** Displacement signal of vibration body, the system conditions are without feedback control and without permanent magnets.

![Displacement signal of vibration body and permanent magnet, the conditions are without feedback control, with permanent magnet and the air-gap of the equilibrium position \(d_0\) is 20 mm](image)

**Fig. 14** Displacement signal of vibration body and permanent magnet, the conditions are without feedback control, with permanent magnet and the air-gap of the equilibrium position \(d_0\) is 20 mm.

Figure 13 and Figure 14 show experimental results of the two conditions without feedback control: one with permanent magnet and the other without permanent magnet. Figure 13 shows the experimental result in the case without permanent magnet, Fig. 14 with permanent magnet and the air-gap of the equilibrium position set to 20 mm. The vibration amplitude and frequency in Fig. 14 are different from those in Fig. 13. Therefore, the vibration body will stop vibrating sooner in the system with permanent magnet than in the system without permanent magnet. However, the system will take a longer time to stop vibrating than with feedback control, as shown in Figs. 15 and 16. When the equilibrium position air-gap is 12.5 mm the vibration body cannot vibrate because the attractive force of the permanent magnet is larger than the spring force of the vibration body.
Fig. 15 Displacement signal of vibration body and permanent magnet of the system, with feedback control and the air-gap of the equilibrium position \( (d_0) \) is 20 mm.

Fig. 16 Displacement signal of vibration body and permanent magnet of the system, with feedback control and the air-gap of the equilibrium position \( (d_0) \) is 12.5 mm.
Figure 15 shows the experimental results with feedback control and equilibrium position of 20 mm. The lower plots are the transient response of the system. From these results, it can be observed that the vibration body stopped vibrating in 3 seconds.

Figure 16 shows the experimental results with feedback control and equilibrium position of 12.5 mm. The lower plots are the transient response of the system. Fig.16 shows the faster damping resulting from greater attractive force. The vibration body stopped vibrating in 1 second.

In the case where the air-gap of equilibrium position is set to 12.5 mm, the vibration body cannot stand still at the equilibrium position and cannot vibrate because the attractive force of the permanent magnet is larger than the spring force of the vibration body.

If the weighting matrices for calculating feedback gain are equal, the vibration body damping time for the air-gap of equilibrium position is set to 12.5 mm, smaller than when it set to 20 mm. It should be noted that eddy current varies with the air-gap setting. These points need further investigation.

In conclusion, the fundamental frequency and time response graphs for the experimental trial are close fit to those from the simulations, and they suggest that the LQR method can be successfully applied in the vibration control system.

7. Conclusions

A vibration suppression control scheme based on linear actuator and permanent magnet is proposed in this paper. The feedback control was designed using the LQR method. In this study, a model was set up for feasibility analysis; the system prototype was manufactured for experimental confirmation. From the results of the simulations and experiments, the following conclusions can be drawn.

(1) According to the model based on analysis of prototype performance, feedback control was achieved by closed-loop stability.

(2) With initial position set to 3 mm, the system with feedback controller could suppress the vibration of the object.

(3) Simulation and experimental results demonstrated the efficiency of the feedback control.

Further research is required to improve the experimental design by applying optimal weighting matrices of the LQR method.

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References


