Frequency Response of Vibration Control with Linear Actuator and Permanent Magnet
Phaisarn SUDWILAI, Koichi OKA and Yuta HIROKAWA

This paper presents the frequency response of the vibration control system. The system prototype comprises a linear actuator, a controller, sensors and permanent magnets. The reduction of vibrations and deformations of thin steel sheets was accomplished by controlling an air-gap between the permanent magnets and the steel sheets. The feedback control systems were designed using the Linear Quadratic Regulator (LQR) method and robust control ($H_{\infty}$). The design of the controller and the frequency responses of the proposed vibration suppression mechanism have been performed through both numerical analyses and experiments. A sinusoidal signal was employed as a continuous disturbance to the system prototype where different conditions of disturbance were carried out by varying the signal frequency from 3 to 60 rad/sec. The frequency responses shown in the Bode diagrams show that the magnitude of vibration body displacement in the case without feedback control is higher than the case with feedback control. In conclusion, the feedback control has been applied successfully, and the proposed vibration suppression mechanism is effective for this vibration control system.

Keywords: vibration control, frequency response, permanent magnet, linear actuator, linear quadratic regulator, $H_{\infty}$.

1. Introduction

In the process of plating, coating and rolling of steel sheets, vibration in conveyances often leads to problems of deformation, peeling and non-uniform products, because the sheets are very flexible. As the results, a vibration suppressor with mechanical contacts is not suitable as a countermeasure. Objects are easily damaged due to their material makeup when they have just been rolled, coated or plated. Therefore, a noncontact suppression mechanism is more suitable for controlling the minimization of deformation of the steel sheets and can minimize such problems. Noncontact vibration control methods which use attractive force of electromagnets have already been proposed (1)-(4). The principal weakness of these methods is that the control range is very constricted, because the attractive force of the magnet varies in inverse proportion to the square of air-gap length. If the vibration amplitude of the object is large, minimizing vibration by means of electromagnets is ineffective for the control of the object. On the other hand, a control method using permanent magnets located at the desired iron sheet passage has also been proposed (5). However, this method uses passive control and can not be successfully suppressed under large disturbances.

A noncontact suspension system using permanent magnets and linear actuators has been proposed (6). The key element of the design is the force control mechanism. A linear actuator drives a permanent magnet and varies the air-gap between the magnet and the object. Variation of the size of the air-gap changes the attractive force. Here the vibration control system uses this type of force control mechanism for vibration suppression, since the control range is almost the same as of the actuator stroke length. In this system vibration control range is expected to be correspondingly wide.

In this paper, vibration suppression mechanism and the frequency response of the vibration control systems of the proposed were studied. The structure of the vibration suppression system is introduced and the aim of the system is shown. The principle of control mechanism is explained and experimental examinations are carried out to demonstrate the frequency response and feasibility of the control method.

2. Vibration Suppression Mechanism

A schematic diagram of a vibration suppression mechanism for plating or coating process is shown in Fig.1. The steel sheet is fed from the right side of the figure and is directed upwards by a roller. While the steel sheet is being fed into the solution bath, plating or coating is carried out. After the plating process is completed, the steel is seasoned or cooled in a vertical feed. During the seasoning process, the steel is especially sensitive to deformation. Consequently vibration control in the seasoning process is very important.

In this paper, a vibration suppression mechanism, as shown in Fig. 1 is proposed. The mechanism consists of two permanent magnets, two linear actuators, sensors which measure the steel sheet displacement, and a controller. The sensors and controller are omitted in the figure. The intention of the system is to reduce vibration caused by the roller feed mechanism, air wipers,
high speed plating. Air wipers, represented by the green element in the figure, adjust the thickness of the plating.

The design of the prototype of vibration suppression mechanism is illustrated in Fig. 2. In this figure, the vertical sheet in the center represents the steel sheet and is labeled “Vibration body”. The magnets on both sides are driven in the horizontal direction by actuators, which are labeled “VCM”. The principle of the vibration suppression method is as follows. When the vibration body swings to the right of the equilibrium position, as shown in Fig. 2 (a1), the left actuator (VCM) drives the left magnet to the right, the displacement between the left magnet and vibration body is reduced, and the left attractive force becomes larger. Consequently, the magnetic force caused the vibration body return to the original equilibrium position. Similarly, when the vibration body swings to the left, as shown in Fig. 2 (b1), the right actuator drives the right magnet to the left, the attractive force on the right side of the vibration body becomes larger and this force caused the object to return to the equilibrium position. The attractive force of the permanent magnet can suppress vibration and deformation of the object.

![Fig. 1 Vibration suppression mechanism](image1)

![Fig. 2 Method of controlling vibration](image2)

3. Experimental Device

An experimental system to examine the performance of the proposed vibration control mechanism was manufactured. The photograph of the system is shown in Fig. 3. The system consists of a vibration body, two permanent magnets installed on a slider, a voice coil motor (VCM) and sensors. The vibration body is structured as a parallel spring made by phosphor bronze and installs steel plates on the both end-sides facing to the permanent magnets. Two permanent magnets are installed on the slider driven by the VCM. Two sensors are installed for measuring the displacements of the vibration body and the slider. A laser sensor is used for the motion of the vibration body and an eddy current sensor is used for the motion of magnets which is the same as the VCM movement.

The control system block diagram is shown in Fig. 3. As seen in the figure, the controller is using a DSP. It calculates the generating force of the VCM from two sensor signals and the output is current signal for the filter. The output of filter is current signal for amplifier. The amplifier changes the signal to the current for the VCM and the VCM changes the currents to the force proportionally. For DSP programming, MATLAB with SIMULINK and Pass/C67 were used.

In this paper, the system was modeled in order to analyze and adopt the linear control theory, to synthesize the control system and frequency response. For modeling, the specifications should be figured out. The relationship between attractive force of permanent magnet, force of vibration body and resultant force with displacement of a permanent magnet and a steel plate on the vibration body set to 20 mm is shown in Fig. 5. As shown in the figure, the relationship of attractive force of permanent magnet and displacement is nonlinear.

Disturbance force was set to sinusoidal function by dc motor drive as shown in dash line in Fig. 3. The relationship between voltage of dc motor and disturbance frequency as shown in Fig. 6.
4. System Modeling

In the model, the motion of the vibration body, as supported by a parallel spring is assumed to be translational. An illustration of the model is shown in Fig. 7. Other specification values of the prototype are indicated in Table 1.

The symbols used in the model are:
- $z_v, z_p$: displacement of the vibration body and permanent magnet,
- $d_0$: air-gap length when the vibration body is centered between the magnets,
- $f_m$: resultant force of magnets,
- $f_{vcm}, f_d$: actuator generating force and the disturbance force.

The resultant force is assumed as inverse proportion to the square of gap length.

$$f_m = \frac{k}{(d_0 - z_v + z_p)^2} - \frac{k}{(d_0 + z_v - z_p)^2}.$$  \hfill (1)

The motion equation of the vibration body is

$$m_v \ddot{z}_v = -k_v z_v - c_v \dot{z}_v + f_m + f_d.$$  \hfill (2)

The motion equation of the slider is

$$m_p \ddot{z}_p = -k_p z_p - c_p \dot{z}_p + f_{vcm} - f_m.$$  \hfill (3)
Table 1 Parameters of the vibration control system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_v ) damping coefficient of the vibration body</td>
<td>0.02 Ns/m</td>
</tr>
<tr>
<td>( \xi_p ) damping coefficient of the permanent magnet</td>
<td>0 Ns/m</td>
</tr>
<tr>
<td>( k_v ) constant of vibration body</td>
<td>150 N/m</td>
</tr>
<tr>
<td>( k_p ) constant of permanent magnet</td>
<td>0 N/m</td>
</tr>
<tr>
<td>( m_v ) mass of vibration body</td>
<td>0.047 kg</td>
</tr>
<tr>
<td>( m_p ) mass of permanent magnet</td>
<td>0.292 kg</td>
</tr>
<tr>
<td>( k_m ) constant of linearization</td>
<td>33.85 N/m</td>
</tr>
</tbody>
</table>

The inputs of the system are defined as the forces of the actuator (VCM) \( f_{vcm} \) and the disturbance \( f_d \). The outputs are the displacement of the vibration body \( z_v \) and the slider \( z_p \). The model can be represented by Eqs. (1), (2) and (3).

By linearization of the attractive force of the magnets, the resultant force can be represented by

\[
\tilde{f}_m = k_v (z_v - z_p),
\]  

(4)

Equations (2) and (3) become as

\[
m_v z''_v = -k_v z_v - c_v z'_v + k_m (z_v - z_p) + f_d,
\]

(5)

\[
m_p z''_p = -k_p z_p - c_p z'_p + f_{vcm} - k_m (z_v - z_p).
\]

(6)

The system can be considered as a block diagram shown in Fig. 6. There are PD controllers in the loops of vibration body and the magnets. The feedback gains are calculated by mean of the LQR control theory. Using the LQR method, a state space model can be derived from Eqs. (4) – (6) as;

\[
\dot{x} = Ax + Bu,
\]

(7)

\[
y = Cx.
\]

(8)

Where, \( u = f_{vcm} \), \( y=[z_v, z_p']^T \), state vector \( x=[z_v, z_p, z'_v, z'_p]^T \).

5. Simulation Results

Simulation results were carried out with nonlinear force. The results of the frequency response of vibration suppression were recorded when the frequency of disturbance are set at 3 and 300 rad/sec. The conditions of the simulations were; 1) without feedback control 2) with feedback control when the controller was designed by LQR method and 3) with feedback control when the controller was designed by robust control (\( H_\infty \)).

The weighting matrices for calculating the optimal gains by LQR method were equal.

![Fig. 9 Displacement signal of vibration body, the system conditions are without feedback control and with feedback control (LQR method), disturbance frequency set to 3 rad/sec.](image)

![Fig. 10 Displacement signal of vibration body, the system conditions are without feedback control and with feedback control (LQR method), disturbance frequency set to 300 rad/sec.](image)

According to the simulation results in case of frequency of disturbance are set to 3 and 300 rad/sec as shown in Figs. 9 and 10. The red line is displacement of vibration body the system without feedback control. The blue line is the system with feedback control. It should be noted that the amplitude of vibration body when the system with feedback control and without feedback control are small different.

Simulation results shows that the feedback control designed by LQR method cannot suppress disturbance.
at low and high frequency. However, in case the controller was designed by robust control ($H_\infty$) the system can suppress disturbance more than when the controller was designed by LQR method, as shown in Figs. 11 and 12.

![Fig. 11 Displacement signal of vibration body, the system conditions are without feedback control and with feedback control ($H_\infty$), disturbance frequency set to 3 rad/sec.](image)

According to simulation results in case of feedback control was designed by robust control ($H_\infty$) as shown in Figs. 11 and 12, all of results proved that the performance of feedback control was designed.

The bode diagram of vibration control system when the conditions were; 1) without feedback control 2) with feedback control (LQR method) and 3) with feedback control ($H_\infty$), as shown in Fig 13.

![Fig. 13 Bode diagram of vibration body, the system conditions are without feedback control and with feedback control (LQR method and $H_\infty$)](image)

6. Experimental Results

Vibration control experiments were also conducted using the experimental prototype shown in Fig. 3. Trial under various conditions for studied in frequency responses such as frequency of disturbance are 3, 36 and 57 rad/sec are shows in Figs. 14, 15 and 16, the feedback control was designed by LQR method. Bode diagram shown in Fig. 17.

![Fig. 14 Displacement signal of vibration body, the system conditions are without feedback control and with feedback control, frequency of disturbance force set to 3 rad/sec.](image)

![Fig. 15 Displacement signal of vibration body, the system conditions are without feedback control and with feedback control, frequency of disturbance force set to 3 rad/sec.](image)
Fig. 16 Displacement signal of vibration body, the system conditions are without feedback control and with feedback control, frequency of disturbance force set to 57 rad/sec.

According to the experimental results in the case of disturbance are set to 3, 36 and 57 rad/sec as shown in Figs. 14, 15 and 16, the blue lines are displacement of vibration body in case of without feedback control. The red lines are displacement of vibration body in case of with feedback control. As the results, in case without feedback control amplitude of vibration body higher than with feedback control. However, the system that design by LQR method cannot suppress disturbance displacement.

Fig. 17 Bode diagram of vibration body, the system condition is with feedback control.

Figure 17 shows bode diagram of vibration body in case of without feedback control and with feedback control was designed by LQR method. From these results, it can be observed that the gains of vibration body in case of with feedback control are lower than without feedback control.

7. Conclusions

The frequency response of vibration suppression control scheme based on linear actuator and permanent magnet is proposed in this paper. The feedback control was designed using the LQR method and robust control ($H_\infty$). A model was set up for feasibility analysis; the system prototype was manufactured for experimental confirmation. From the results of the experiments, according to the model based on analysis of prototype performance, feedback control were achieved by closed-loop stability and demonstrated the efficiency of the feedback control.

Further research is required to improve the experimental design by applying robust control ($H_\infty$).

References


