Measurements of negative and complex probabilities with photon polarization

Y. Suzuki, *et. al.*, *New Journal of Physics* **14** (2012) 103022

Masataka Iinuma

Hiroshima University,

Graduate school of Advanced Sciences of Matters

Quantum Frontier Group

Seminar in Kochi Institute of Technology

Quantum Frontier Group

Joint research group

High Energy Physics lab.

Quantum Optics Lab.

Explore fundamental aspect of quantum mechanics based on optics

http://home.hiroshima-u.ac.jp/qfg/qfg/index.html

Outline

0. Summary

1. Quantum state tomography of photon polarization

2. Statistical approach in quantum mechanics

3. Sequential measurement of non-commuting observables

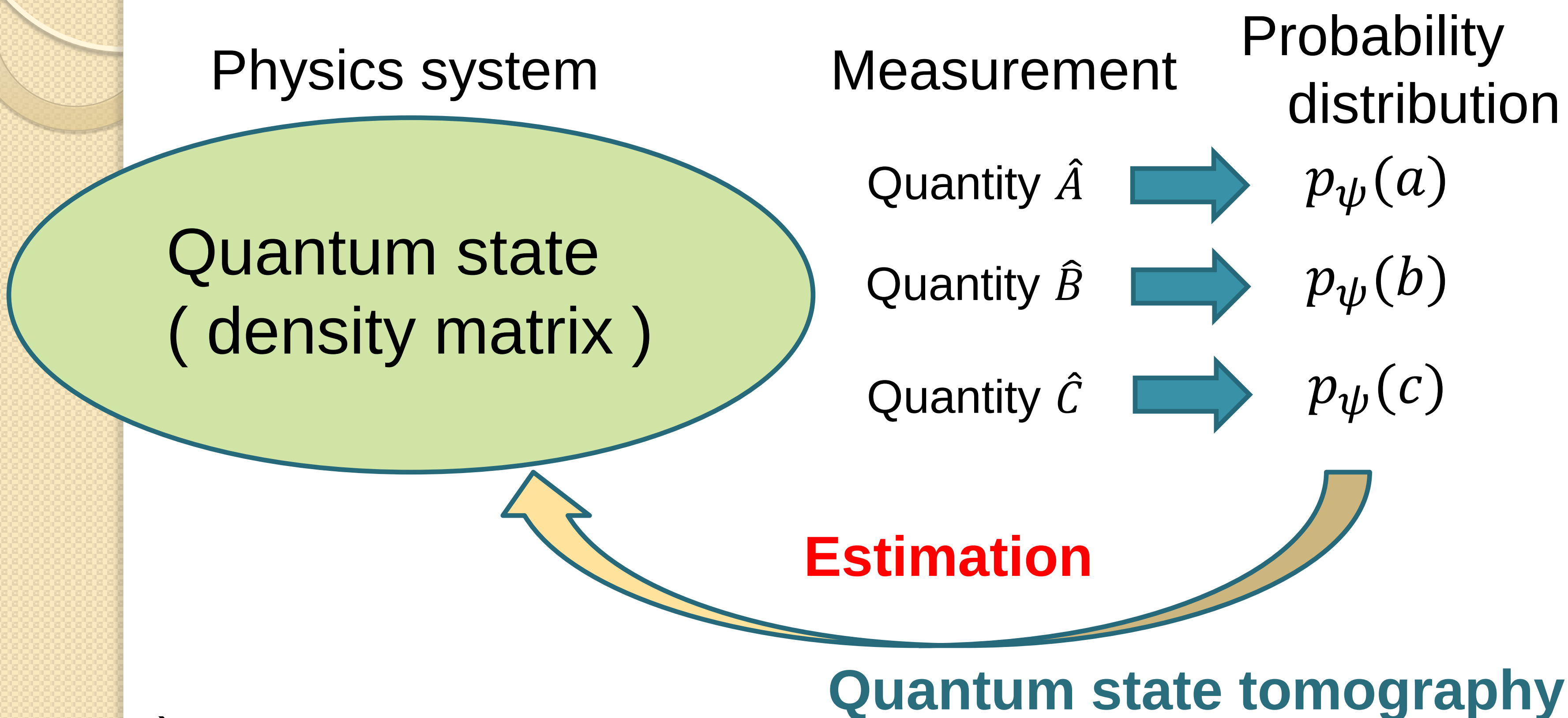
4. Summary, intriguing questions, and perspective

Summary

- It is natural that quantum state is expressed by an negative or complex joint probability distribution.
- Experimentally-obtained probabilities are never identified to the probabilities before a measurement due to the interaction to the meter apparatus. Intrinsic probabilities are converted to positive probabilities by the measurement interaction.
- The results of weak measurement shows the intrinsic probability before the measurement process.

1. Quantum state tomography of photon polarization

Quantum state tomography



Ex)

- Quantum information processing
- Electronic state and spin state in molecules, solid, etc.

Quantum information processing

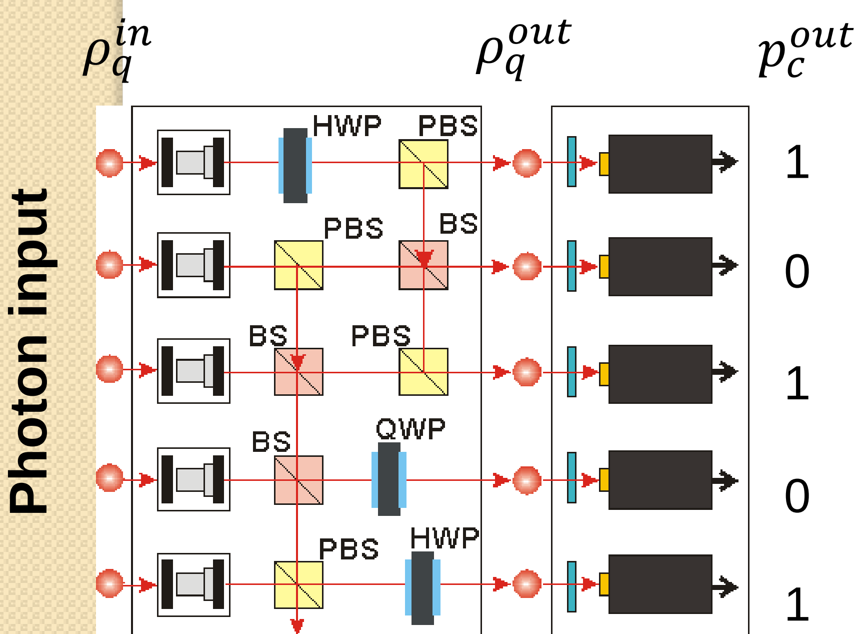
Quantum state: ρ_q Operation : Unitary transformation

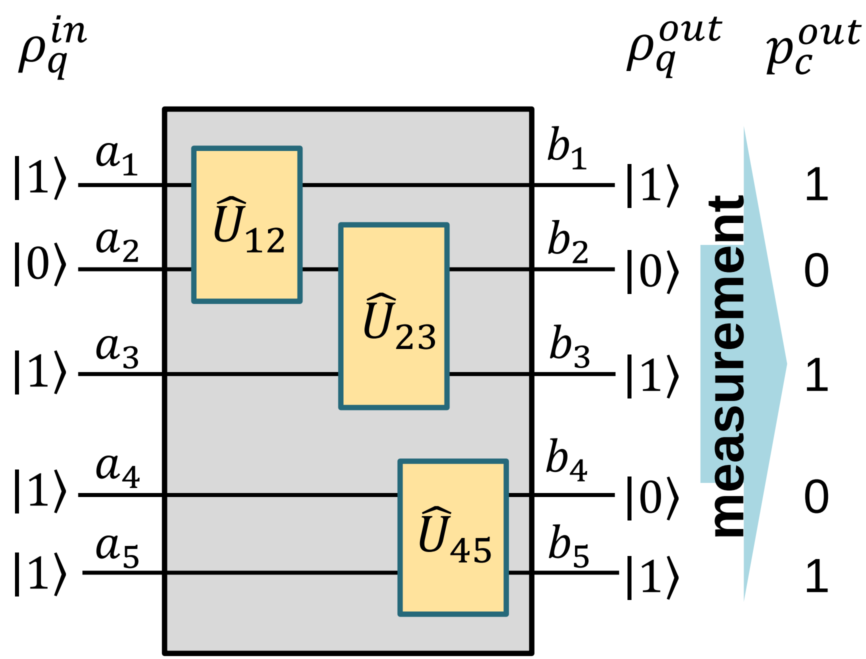
processing

$$\rho_q^{in}(a_1, a_2, a_3, a_4, a_5, \cdots) \longrightarrow \rho_q^{out}(b_1, b_2, b_3, b_4, b_5, \cdots)$$

measurement

$$\rho_q^{out}(b_1, b_2, b_3, b_4, b_5, \cdots) \longrightarrow p_c^{out}(b_1, b_2, b_3, b_4, b_5, \cdots)$$





Density matrix in a two-level system

General representation of Quantum state

Matrix with two bases $|0\rangle$ and $|1\rangle$ in Hilbert space (pure state)

$$\hat{\rho} \equiv |\psi\rangle\langle\psi| = \begin{pmatrix} |C_0|^2 & C_1^* C_0 \\ C_0^* C_1 & |C_1|^2 \end{pmatrix} = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \qquad \begin{matrix} \rho_{00} + \rho_{11} = 1 \\ \rho_{01}^* = \rho_{10} \end{matrix}$$

$\hat{\rho}$ should be a Hermitian matrix with its trace one.

ρ_{00} and ρ_{11} : Probability of $|0\rangle$ and $|1\rangle$

Ex)

Mixed state

Probability of $|0\rangle$: 1/2

Probability of $|1\rangle$: 1/2

$$\hat{\rho}_{mix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

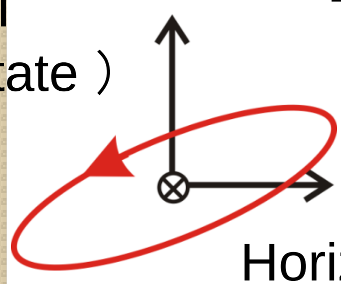
Superposition state

$$|\psi\rangle = 1/\sqrt{2}\,(|0\rangle + |1\rangle)$$

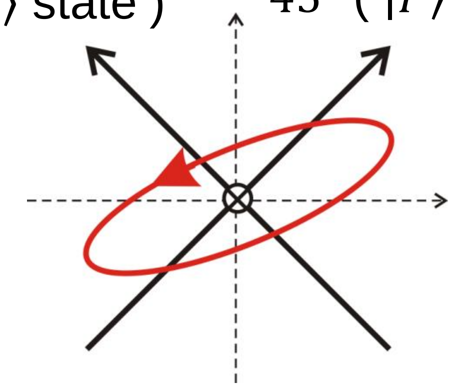
$$\hat{\rho}_s = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Basis of polarization state

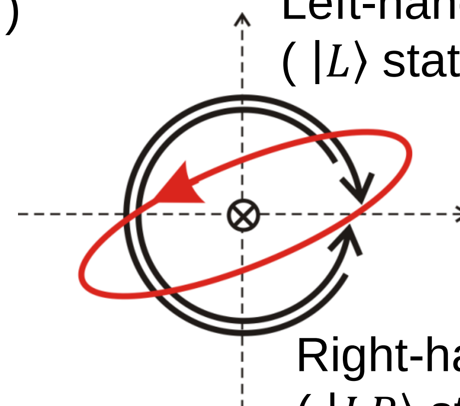
HV basis



PM basis



LR basis



$$|\psi\rangle = C_H|H\rangle + C_V|V\rangle$$

$$|\psi\rangle = C_P|P\rangle + C_M|M\rangle$$

$$|P\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$$

$$|M\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$$

$$|\psi\rangle = C_L|L\rangle + C_R|R\rangle$$

$$|L\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle)$$

$$|R\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i|V\rangle)$$

Observables of photon polarization

In two-level systems

Two eigenvalues $A_m = \pm 1 \quad m = \pm 1$

Two eigenstates : measurement bases $|m\rangle$

Observable : $\hat{A} = \sum_{m=\pm 1} A_m |m\rangle\langle m|$

HV basis $\hat{S}_{HV} = |H\rangle\langle H| - |V\rangle\langle V| = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \hat{\sigma}_z$

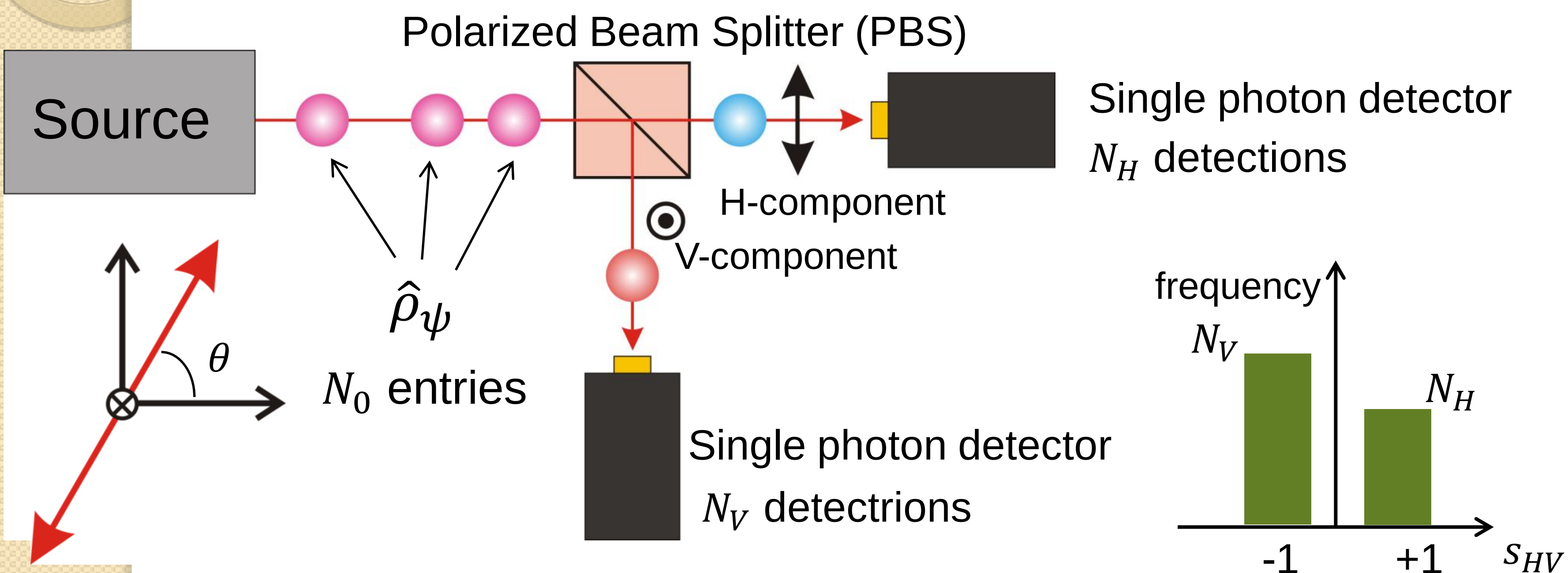
PM basis $\hat{S}_{PM} = |P\rangle\langle P| - |M\rangle\langle M| = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \hat{\sigma}_x$

LR basis $\hat{S}_{LR} = |L\rangle\langle L| - |R\rangle\langle R| = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \hat{\sigma}_y$

Pauli matrices

Measurements of polarization

Measurement of HV basis ($\hat{S}_{HV}$ measurement)



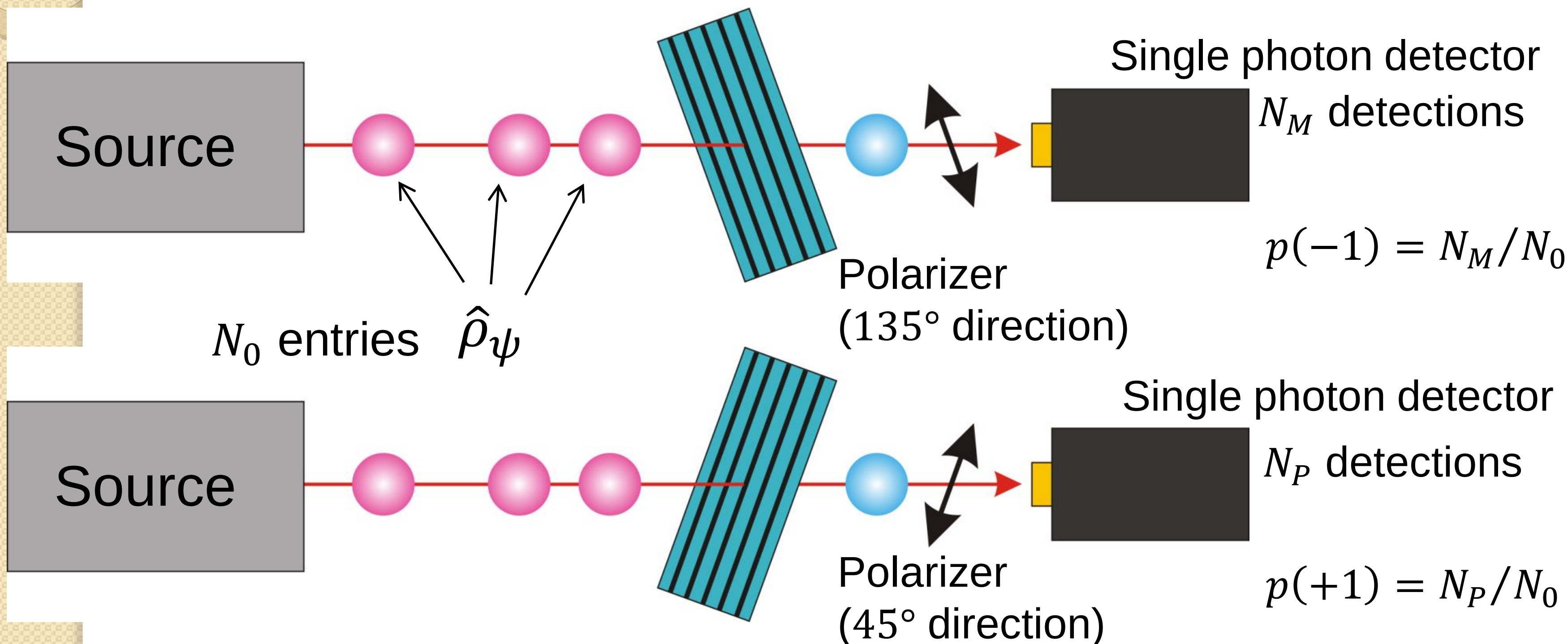
$$p(+1) = |\cos\theta|^2 = N_H/N_0$$
$$p(-1) = \underline{|\sin\theta|^2} = N_V/N_0$$

$$\hat{\rho}_\psi = \begin{bmatrix} p(+1) & \rho_{01} \\ \rho_{01}^* & p(-1) \end{bmatrix}$$

Off-diagonal components can be not fixed.

Measurement of polarization

Measurement of PM basis ($\hat{S}_{PM}$ measurement)



$$x \equiv \langle\psi|\hat{S}_{PM}|\psi\rangle = p(+1) - p(-1) \quad \Rightarrow \quad \hat{\rho}_\psi = \begin{bmatrix} \rho_{00} & x \\ x & \rho_{11} \end{bmatrix}$$

Determination of only real parts of off-diagonal

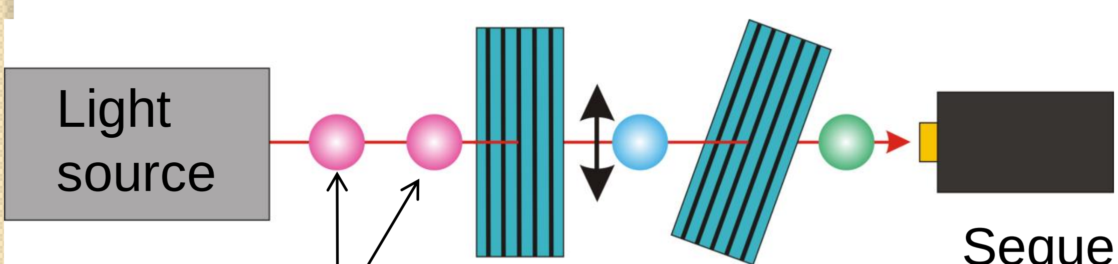
Non-commuting observables

$\hat{A}$ & $\hat{B}$ non-commuting
$\left[\hat{A},\hat{B}\right] = \hat{A}\hat{B} - \hat{B}\hat{A} \neq 0$
$\Longleftrightarrow$
Never fix precise values
of $\hat{A}$ & $\hat{B}$ (uncertainty principle)

Observables of polarization $\hat{S}_{HV}$ & $\hat{S}_{PM}$

$\left[\hat{S}_{HV},\hat{S}_{PM}\right] = i\hat{S}_{LR}$ $\Longleftrightarrow$ $s_{HV} = +1$（H polarization）
$s_{PM} = +1$ or -1 (not fixed)

Trade-off relation



Sequential measurement of
$\hat{S}_{HV}$ & $\hat{S}_{PM}$: **No Work !!**

Estimation of density matrix

Density matrix $\hat{\rho}_\psi$: Never directly obtainable in experiments

Estimation of a single system

$$\hat{\rho}_\psi = \frac{1}{2}\left(\hat{I} + x\hat{\sigma}_x + y\hat{\sigma}_y + z\hat{\sigma}_z\right) = \frac{1}{2}\begin{bmatrix} 1+z & x-iy \\ x+iy & 1-z \end{bmatrix}$$

$$x = Tr\left[\underline{\hat{\rho}_\psi} \cdot \underline{\hat{\sigma}_x}\right] = \langle \hat{\sigma}_x \rangle$$

Estimation from averages of observables $\hat{\sigma}_x$、$\hat{\sigma}_y$、$\hat{\sigma}_z$

Physical quantity

No directly obtainable

Ex) polarization of single photon

$$z = \langle \hat{S}_{HV} \rangle = p(H) - p(V)$$

$$x = \langle \hat{S}_{PM} \rangle = p(P) - p(M)$$

$$y = \langle \hat{S}_{LR} \rangle = P(L) - P(R)$$

Three unknown parameters
In simple case, measurements with 6 bases
At least, measurements with 4 bases

2. Statistical approach to quantum mechanics

Notation

Conditional probability

$$p(\underbrace{m}_{\text{Random variables}} \,|\, \underbrace{a}_{\text{condition}})$$

Probability of *m* under the condition *a*

Conditional joint probability

$$p(\underbrace{m, f}_{\text{Random variables}} \,|\, \underbrace{a}_{\text{condition}})$$

Probability of (*m*, *f*) under the condition *a*

Probability of *a* in the initial state ψ

$$p(a \,|\, \psi)$$

Pseudo-probability distribution

Expression of quantum state as joint probability distribution with variables of non-commuting observables

Joint probability : $p(a,b|\psi)$ $\Longleftrightarrow$ Density matrix : $\hat{\rho}_\psi$

Ex) probability distribution $p(x,p)$

Wigner distribution (1932 E.P. Wigner)

$$W_\psi(x,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\langle x+\frac{1}{2}x'\middle|\hat{\rho}_\psi\middle|x-\frac{1}{2}x'\right\rangle e^{-ipx'}dx'$$

Kirkwood-Dirac distribution (1933 J.G. Kirkwood, 1944 P. Dirac)

$$K_\psi(x,p) = \langle x|p\rangle\langle p|\hat{\rho}_\psi|x\rangle$$

Generally, negative and complex number $\Longleftrightarrow$ Never directly measurable

Kirkwood-Dirac distribution

Joint probability distribution on $\hat{A}$、$\hat{B}$

$$p(a,b|\psi) = \langle b|a\rangle\langle a|\hat{\rho}_\psi|b\rangle \quad [\ = \langle b|a\rangle\langle a|\psi\rangle\langle\psi|b\rangle\ ]$$

pure state

$$\hat{A} = \sum_a A_a|a\rangle\langle a| \quad \hat{B} = \sum_b B_b|b\rangle\langle b|$$

Generally, values are complex numbers.

Open questions of consistency with actual measurement results (positive probabilities)

Giving correct marginal probabilities

$$p(a) = \sum_b p(a,b|\psi) = \langle a|\hat{\rho}_\psi|a\rangle \quad p(b) = \sum_a p(a,b|\psi) = \langle b|\hat{\rho}_\psi|b\rangle$$

Including a correlation (commutation relation)

$$[\hat{A},\hat{B}] = i\hat{C} \quad \langle\hat{C}\rangle = \frac{1}{i}\langle[\hat{A},\hat{B}]\rangle = 2\Im[\langle\hat{A}\hat{B}\rangle] = 2\Im\left[\sum_{a,b} A_a B_b p(a,b|\psi)\right]$$

Correspondence to density matrix

$$\hat{\rho}_\psi = \sum_{a,b} |a\rangle\langle a|\hat{\rho}_\psi|b\rangle\langle b| = \sum_{a,b} \langle b|a\rangle\langle a|\hat{\rho}_\psi|b\rangle \frac{|a\rangle\langle b|}{\langle b|a\rangle}$$

$$= \sum_{a,b} \underbrace{p_\psi(a,b|\psi)}_{\text{K-D distribution}} \frac{|a\rangle\langle b|}{\langle b|a\rangle}$$

The K-D distribution is identified to each component of a density matrix using $|a\rangle\langle b|/\langle b|a\rangle$ as a basis.

Mathematically,

K-D distribution = density matrix

Bayes' theorem

$$p(a,b|\psi) = \underline{p(a|b,\psi)}\ \underline{p(b|\psi)}$$

Joint probability　Conditional probability　Transition probability

Assuming K-D distribution as a joint probability,

$$p(a,b|\psi) = p(a|b,\psi)\,\underbrace{p(b)}_{|\langle b|\psi\rangle|^2} = \langle b|a\rangle\langle a|\psi\rangle\langle\psi|b\rangle$$

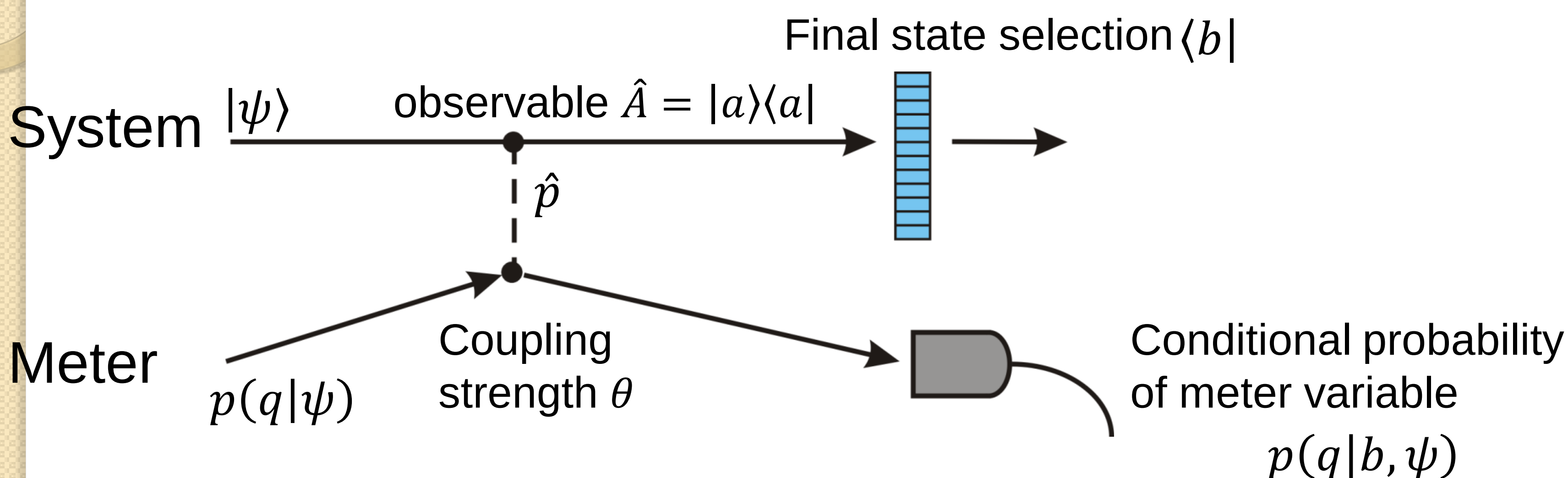
Conditional probability:

$$p(a|b,\psi) = \frac{\langle b|a\rangle\langle a|\psi\rangle}{\langle b|\psi\rangle}$$

Weak value
initial state : $|\psi\rangle$, final state: $\langle b|$、
projection operator : $\hat{A} = |a\rangle\langle a|$

How to reconstruct K-D distribution

Use of weak measurement



Back-action : negligible ⇒ Weak value : $\langle \hat{A} \rangle_w = \dfrac{\langle b|a\rangle\langle a|\psi\rangle}{\langle b|\psi\rangle}$

K-D distribution : $p(a, b|\psi) = \langle \hat{A} \rangle_w \, p(b)$

Position and momentum : C. Bamber and J. S. Lundeen, PRL Vol. 112 070405 (2014)
Photon polarization : J. Z. Salvail, et. al. Nature Photon Vol. 7 316-321 (2013)

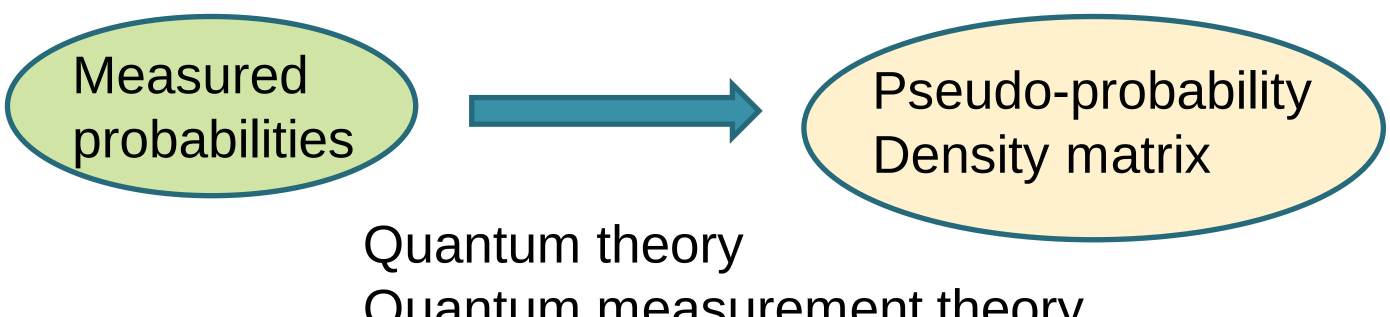
Questions

- If the K-D distribution represents a quantum state, it should be independent in the measurement process.

  -> Can we obtain the K-D distribution in the strong measurement ?

- The K-D distribution is just one of mathematical representations such as pseudo-probability distributions.
  (Ex : Winger distribution, Q function, etc..)

  -> Can we get the probability distribution without the help of quantum theory (or quantum measurement theory) ?



3. Sequential measurement of photon polarization

Alternative approach

Measurements at any measurement strength

Analysis without the help of quantum theory

Probability distribution

$\rho_\psi(a, b, \cdots)$

Analysis

Without the use of quantum theory

measurement $(\hat{A}, \hat{B}, \cdots)$

$\hat{\Pi}_{\hat{A}}, \hat{\Pi}_{\hat{B}}, \cdots$

$p_{exp}(a) \quad p_{exp}(b)$

Comparison to each other

Probabilities as relative frequency

Quantum measurement theory

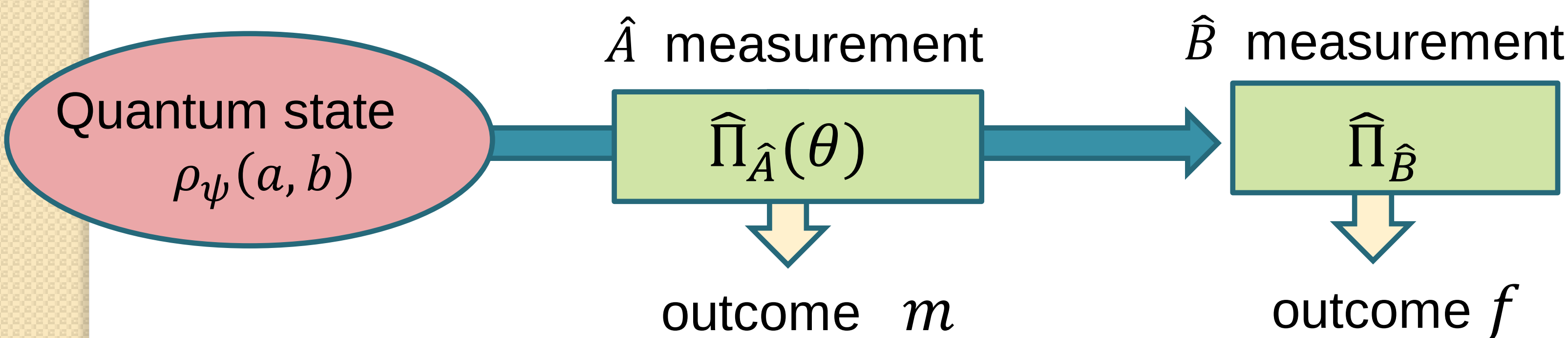
Quantum theory

What is obtained as probability distributions ?

Sequential measurement

Use of Variable Strength Measurement (VSM)

$\widehat{\Pi}_{\hat{A}}(\theta)$ Measurement strength θ : controllable $\quad [\hat{A}, \hat{B}] = i\hat{C}$



Experimental joint probability $p_{exp}(m, f|\theta)$

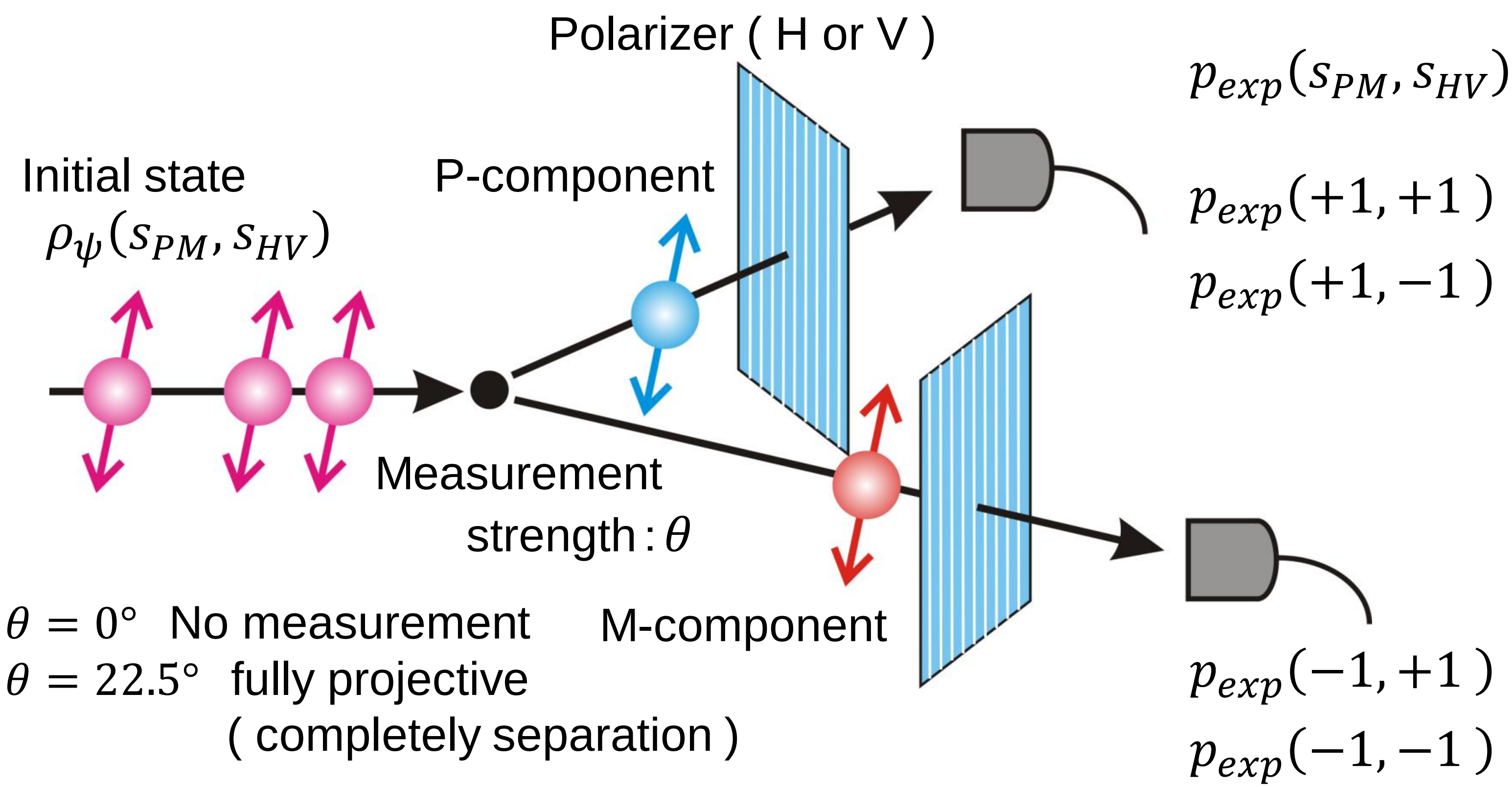
$$p_{exp}(m, f|\theta) = \sum_{a.b} \underline{p(m, f|a, b, \theta)}\,\underline{\rho_\psi(a, b\,)}$$

Experimental joint probability

Error probability by meter apparatus

joint probability of initial state

Sequential measurements of $\hat{S}_{PM}$ and $\hat{S}_{HV}$

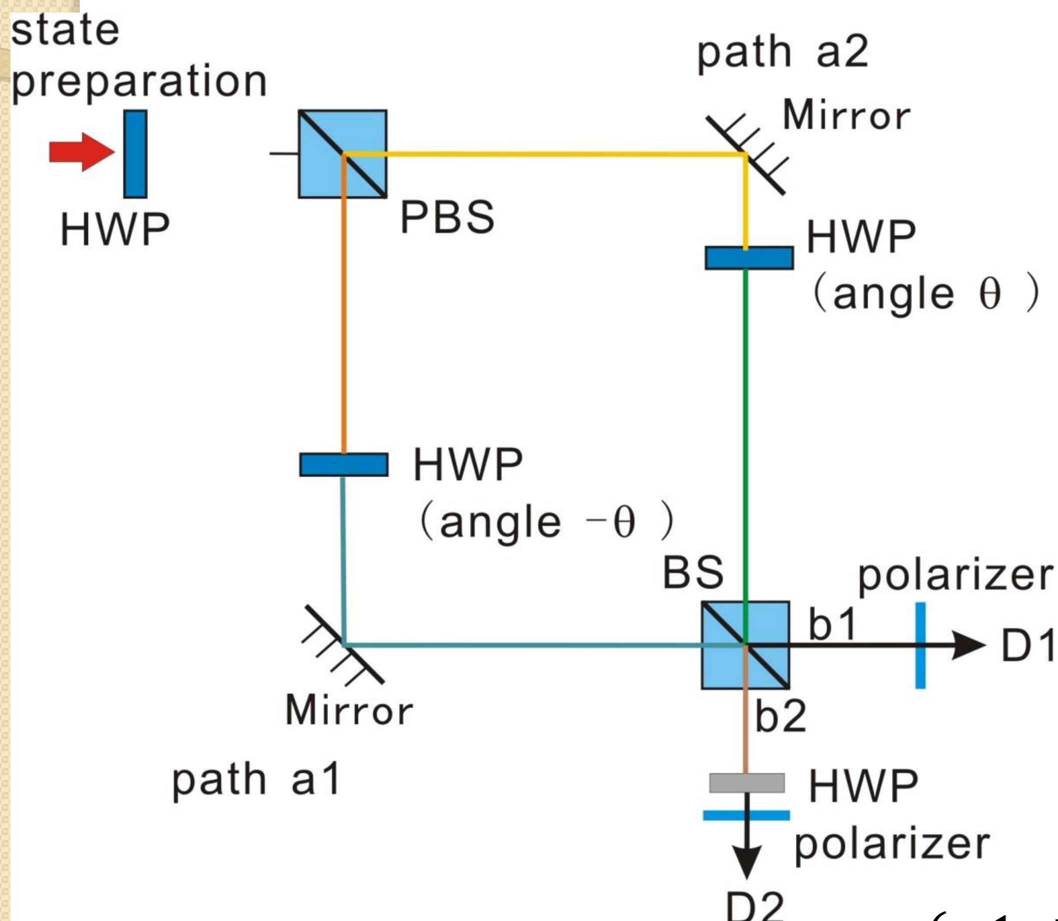


$p_{exp}(s_{PM}, s_{HV})$ depends on the measurement strength θ

$\rho_\psi(s_{PM}, s_{HV})$ should be independent in the measurement strength θ

Setup for linear polarization

Initial state : $|\psi_i\rangle = \sin\phi_1|H\rangle + \cos\phi_2|V\rangle$



Measurement operator

$$\hat{M}_P = \frac{1}{\sqrt{2}}\left[\cos 2\theta + \sin 2\theta\, \hat{S}_{PM}\right]$$

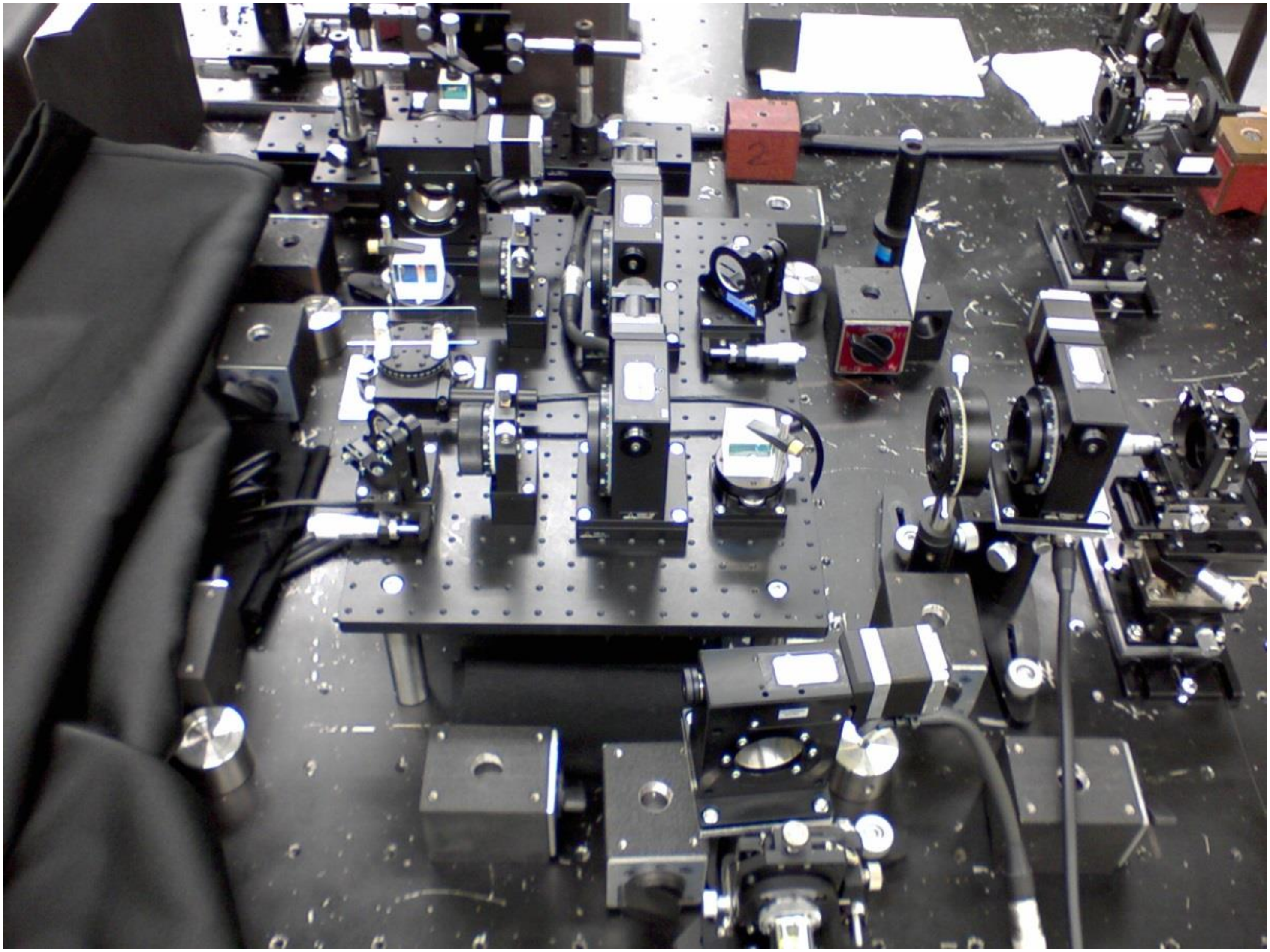
$$\hat{M}_M = \frac{1}{\sqrt{2}}\left[\cos 2\theta - \sin 2\theta\, \hat{S}_{PM}\right]$$

$p_{exp}(+1,+1)$

$p_{exp}(+1,-1)$

$p_{exp}(-1,+1)$

$p_{exp}(-1,-1)$



Resolution and Back-action

Influence of $\hat{S}_{PM}$ measurement

Measurement resolution ε

Capability of separation
between P and M

$\varepsilon = 0$ no separation
$\varepsilon = 1$ perfect separation

Inputting P state as initial state

$$\varepsilon = p_{exp}(P|P) - p_{exp}(M|P)$$

Probability of M in inputting P
(probability of P in inputting M)

$$p_{PM} = \frac{1}{2}(1-\varepsilon)$$

Error probability by resolution

Measurement back-action η

Mixing by flipping
between H and V

$\eta = 0$ no mixing
$\eta = 1$ perfect mixing

Inputting H state as initial state

$$1-\eta = p_{exp}(H|H) - p_{exp}(V|H)$$

Probability of V in inputting H
(probability of H in inputting V)

$$p_{HV} = \frac{\eta}{2}$$

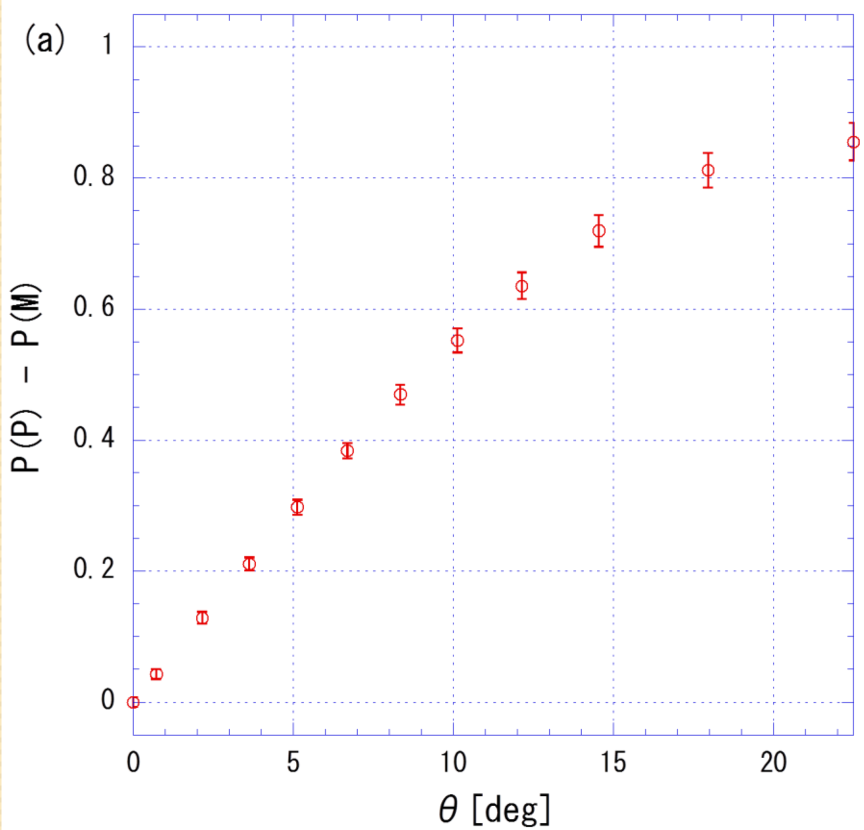
Error probability by back-action

Evaluation of ε and η

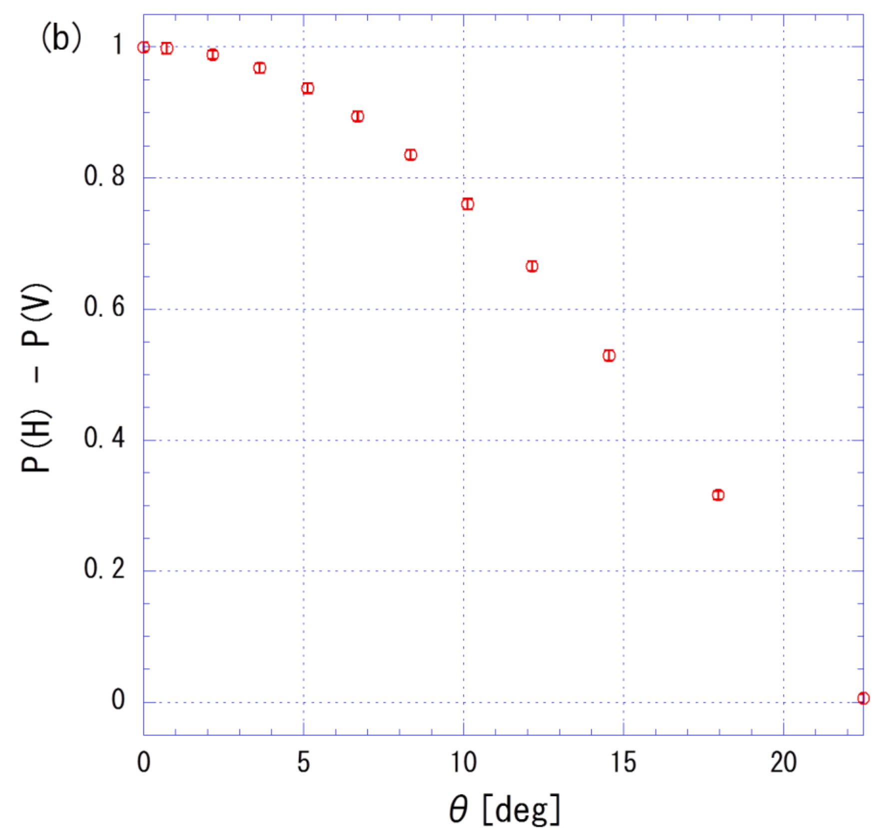
Measurement resolution ε

back-action $1-\eta$

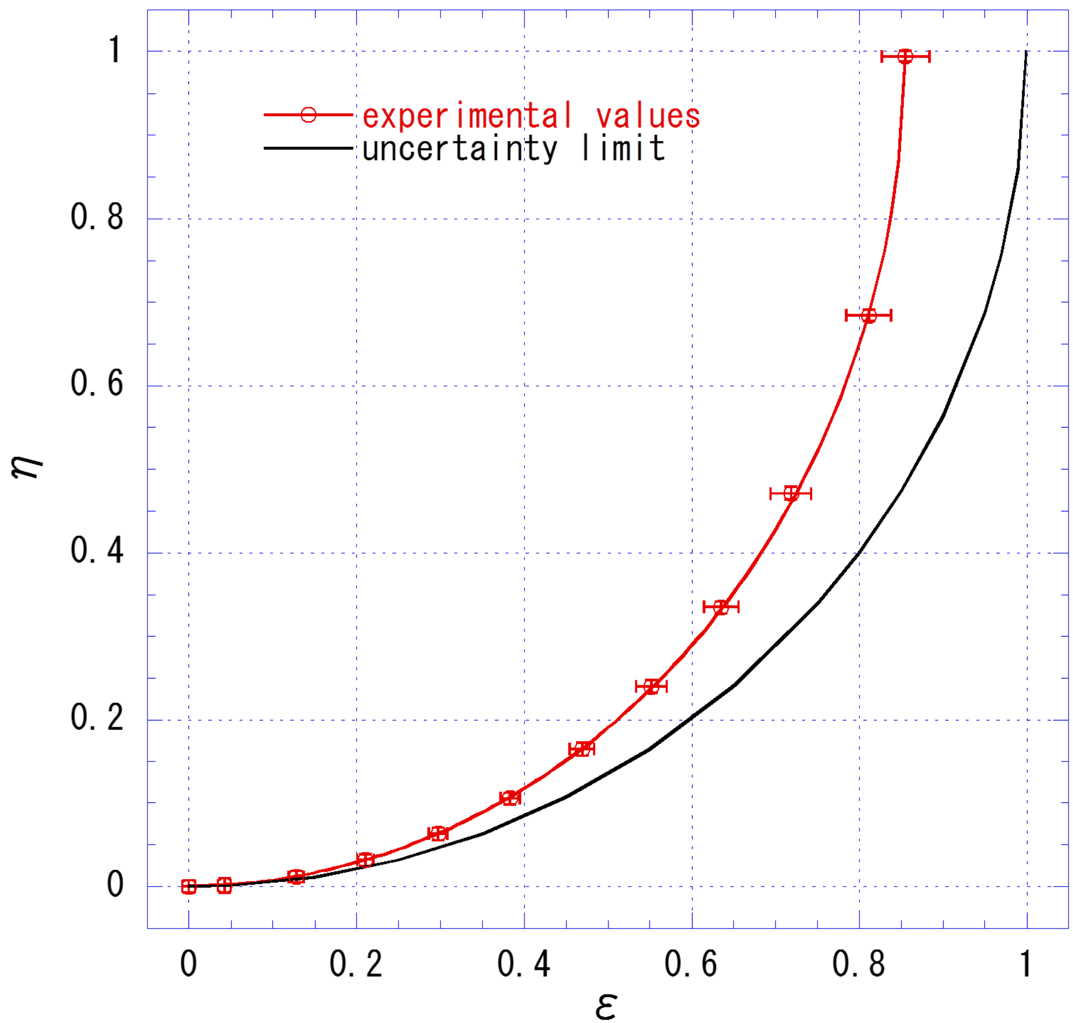
Initial state : P state



Initial state : H state



Measurement uncertainty



$$\varepsilon = V_{PM} \sin 4\theta$$

$$\eta = 1 - V_{HV} \cos 4\theta$$

Experimental imperfection

$V_{PM} = 85.3\ \%$

$V_{HV} = 99.97\ \%$

Ozawa's error and disturbance
(independent in initial state)

$$\varepsilon_o^2 = 4p_{PM} = 2(1-\varepsilon)$$

$$\eta_o^2 = 4p_{HV} = 2\eta$$

Analysis (for linear polarization)

Resolution : ε Error probability by resolution : $p_{PM} = \frac{1}{2}(1-\varepsilon)$

Back-action : η Error probability by back-action : $p_{HV} = \frac{\eta}{2}$

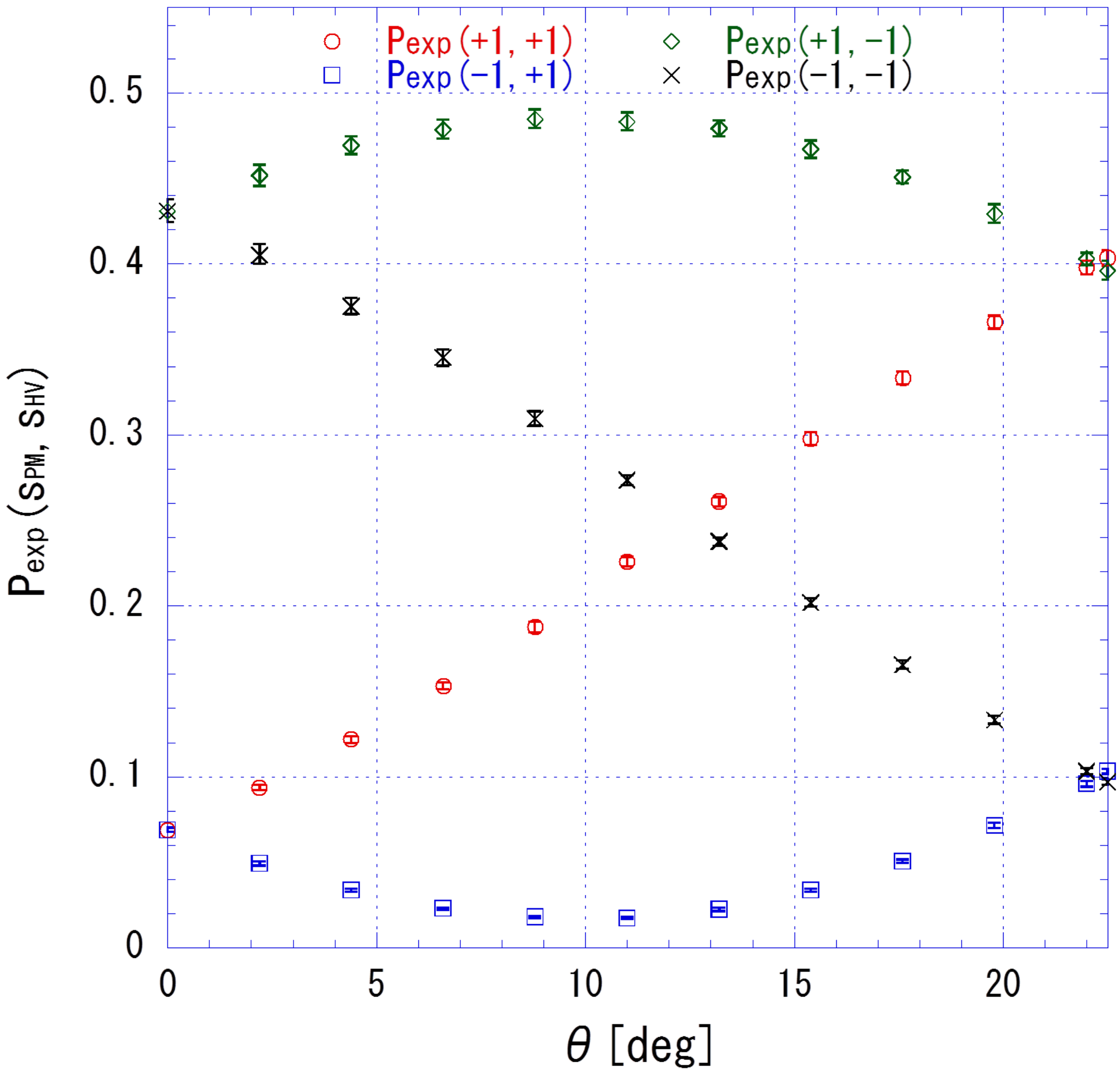
Initial state : $\rho_\psi(s_{PM}, s_{HV})$ Measured probability : $p_{exp}(s_{PM}, s_{HV})$

$$p_{exp}(s_{PM}, s_{HV}) = \frac{1+\varepsilon}{2}\left(1-\frac{\eta}{2}\right)\underbrace{\rho_\psi(s_{PM}, s_{HV})}_{\text{Initial probability}} + \frac{1-\varepsilon}{2}\left(1-\frac{\eta}{2}\right)\underbrace{\rho_\psi(-s_{PM}, s_{HV})}_{\text{Only } s_{PM} \text{ flip}}$$

$$+\left(\frac{1+\varepsilon}{2}\right)\frac{\eta}{2}\underbrace{\rho_\psi(s_{PM}, -s_{HV})}_{\text{Only } s_{HV} \text{ flip}} + \left(\frac{1-\varepsilon}{2}\right)\frac{\eta}{2}\underbrace{\rho_\psi(-s_{PM}, -s_{HV})}_{\text{Both of } s_{PM} \text{ and } s_{HV} \text{ flip}}$$

No use of quantum theory and quantum measurement theory

Experimental joint probabilities



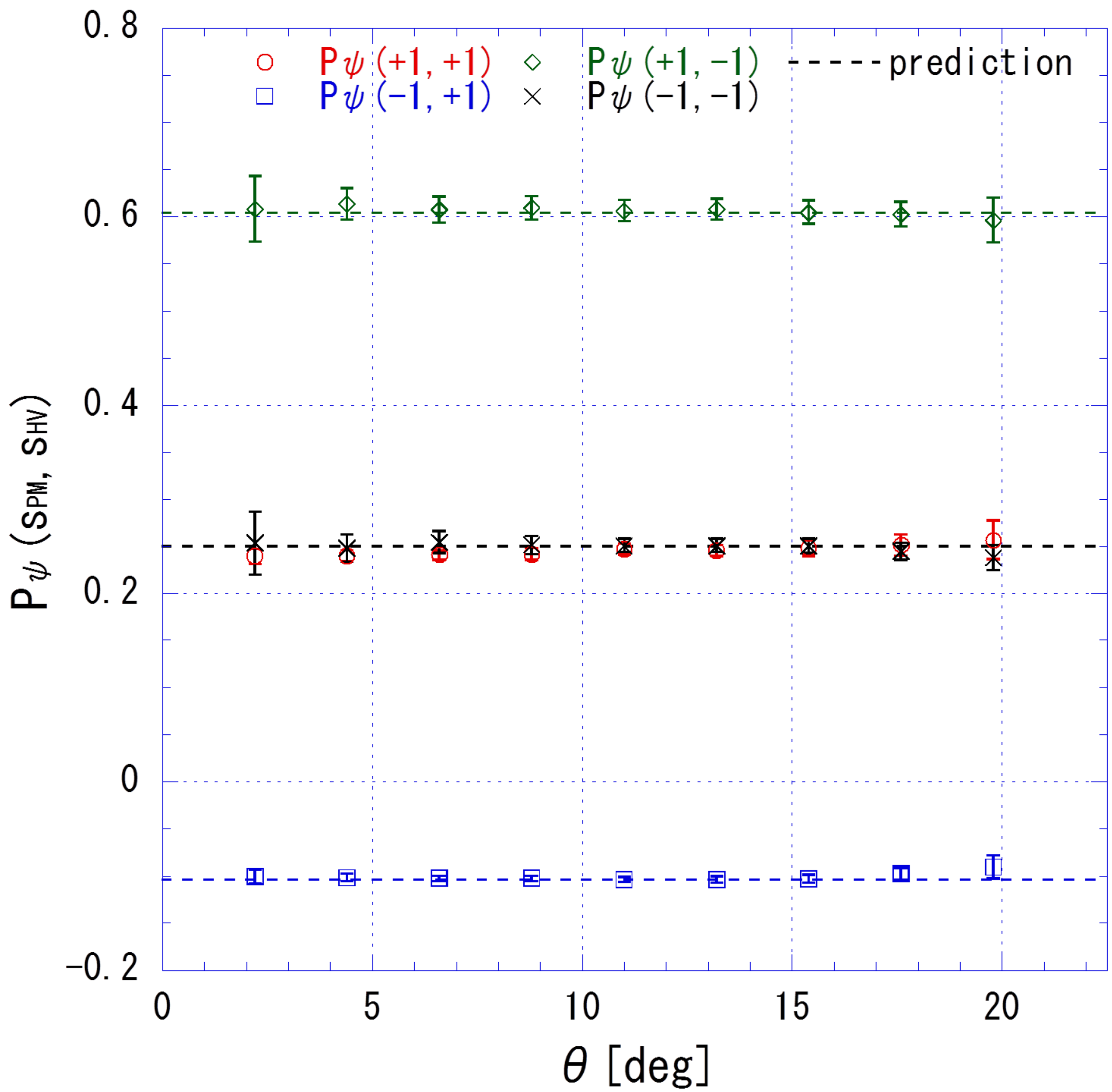
Initial state :

$$|\psi\rangle = \begin{bmatrix}\cos\phi \\ \sin\phi\end{bmatrix} \quad \phi = 67.5°$$

$p_{exp}(s_{PM}, s_{HV})$
depending on θ

Including the influence of $\hat{S}_{PM}$ measurement

Reconstruction of joint probabilities



Y. Suzuki, *et. al.*, *New Journal of Physics* **14** (2012) 103022

$p_\psi(s_{PM}, s_{HV})$
Independent in θ

Including no influence of the $\hat{S}_{PM}$ measurement

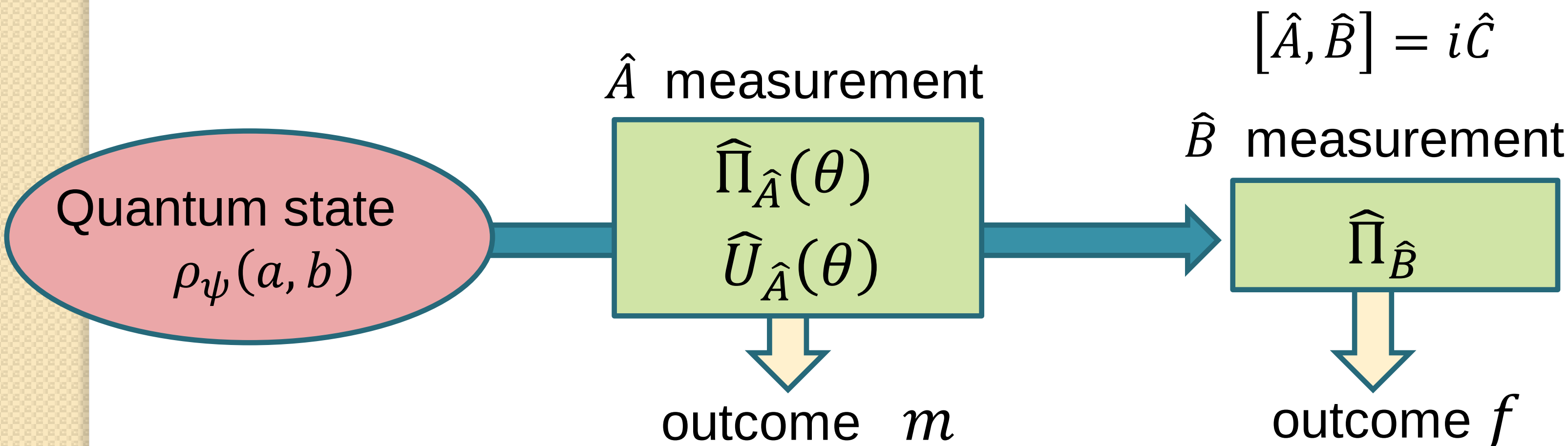
Consistent with K-D distribution
$\langle s_{HV}|s_{PM}\rangle\langle s_{PM}|\rho_\psi|s_{HV}\rangle$

One of joint probabilities is negative !!

Measurement of complex probability

Measurement of statistical property : projective process
Measurement of dynamical property : unitary process

H. F. Hofmann, *New J. Phys.* **13** 103009 (2011)



Experimental joint probability $p_{exp}(m, f|\theta)$

$$p_{exp}(m, f|\theta) = \sum_{a.b} \underline{p(m, f|a, b, \theta)}\,\underline{\rho_\psi(a, b)}$$

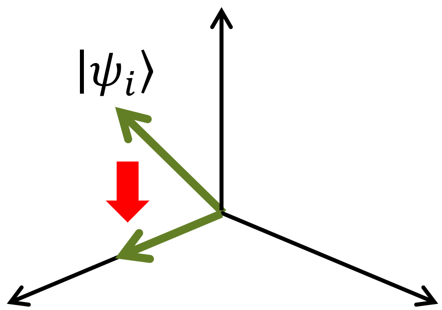
Experimentally joint probability

Error probability (complex number)

Intrinsic joint probability (complex number)

Measurement process

Projection process

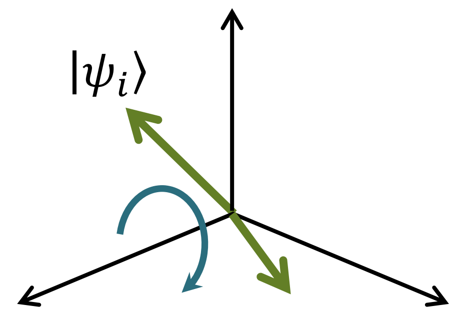


PM axis

$$\hat{P}_P = \frac{1}{\sqrt{2}}\left[\cos 2\theta + \sin 2\theta\,\hat{S}_{PM}\right]$$

$$\hat{P}_M = \frac{1}{\sqrt{2}}\left[\cos 2\theta - \sin 2\theta\,\hat{S}_{PM}\right]$$

Unitary process



PM axis

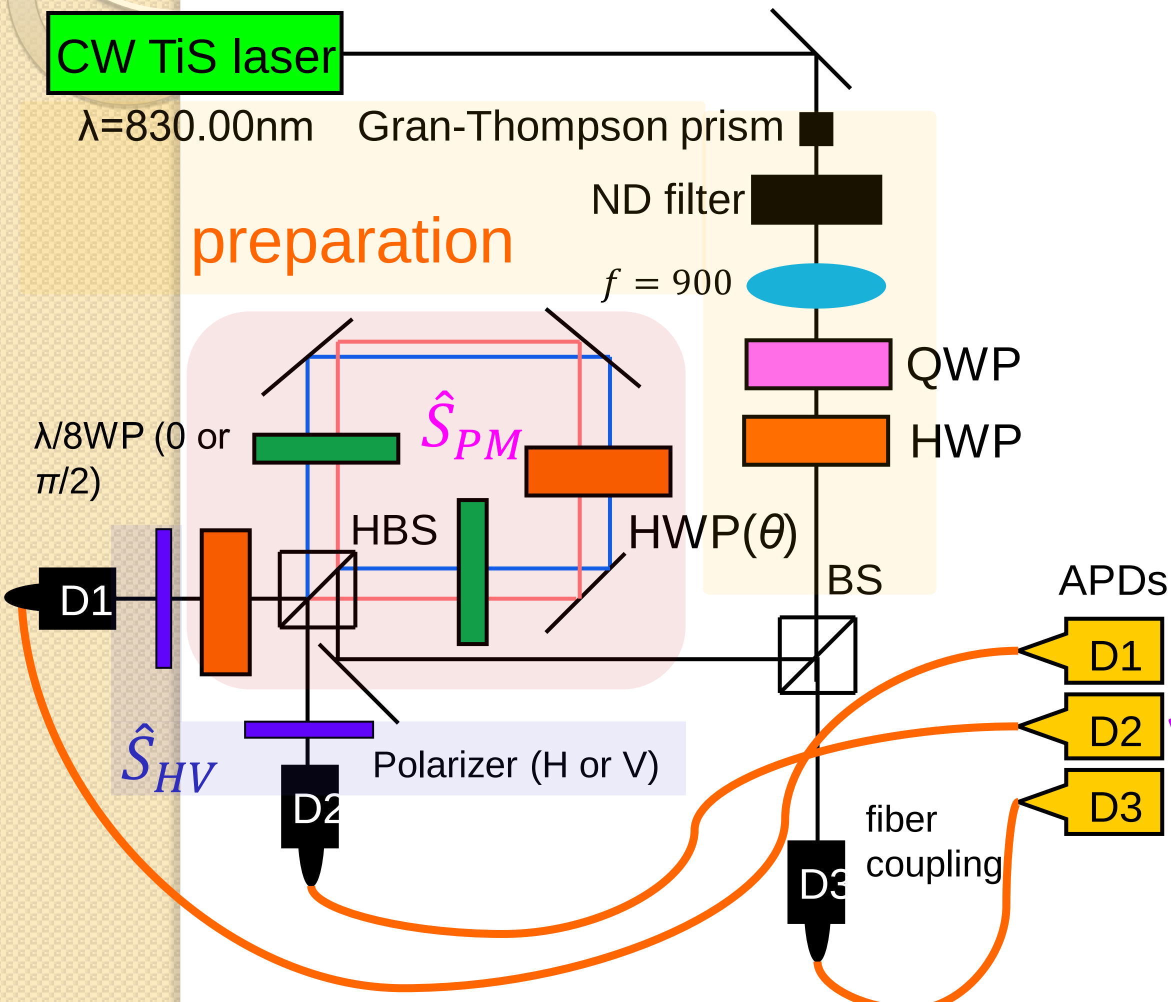
$$\hat{U}_P = \frac{1}{\sqrt{2}}\left[\cos 2\theta + i\sin 2\theta\,\hat{S}_{PM}\right]$$

$$\hat{U}_M = \frac{1}{\sqrt{2}}\left[\cos 2\theta - i\sin 2\theta\,\hat{S}_{PM}\right]$$

unify

$$\hat{M}_P = \frac{1}{\sqrt{2}}\left[\cos 2\theta + e^{i\varphi}\sin 2\theta\,\hat{S}_{PM}\right] \quad \hat{M}_M = \frac{1}{\sqrt{2}}\left[\cos 2\theta - e^{i\varphi}\sin 2\theta\,\hat{S}_{PM}\right]$$

Setup for complex probability



Measurement strength : q

$\theta = 0°$: no measurement

$\theta = 22.5°$: fully projection
for $\hat{S}_{PM}$
full resolution
for correlation

$p_{exp}(+1,+1)$ or $p_{exp}(+1,-1)$
[P, H] [P, V]

$p_{exp}(-1,+1)$ or $p_{exp}(-1,-1)$
[M, H] [M, V]

Definition of resolution and back-action

$$\eta \equiv \frac{\sum_{s_{PM},s_{HV}} s_{HV} p_{exp}(s_{PM}, s_{HV})}{\langle\psi_i|\hat{S}_{HV}|\psi_i\rangle}$$

Back-action of $\hat{S}_{PM}$ measurement

$$\varepsilon_r \equiv \frac{\sum_{s_{PM},s_{HV}} s_{PM} p_{exp}(s_{PM}, s_{HV})}{\langle\psi_i|\hat{S}_{PM}|\psi_i\rangle}$$

Resolution of $\hat{S}_{PM}$ measurement

$$\varepsilon_i \equiv i\,\frac{\sum_{s_{PM},s_{HV}} s_{PM} s_{HV} p_{exp}(s_{PM}, s_{HV})}{\langle\psi_i|\hat{S}_{HV}\hat{S}_{PM}|\psi_i\rangle}$$

Resolution of correlation

Calibration of meter system

Input of H-polarization at θ

back-action by $\hat{S}_{PM}$ measurement

$$\eta = \left(p_{exp}(P,H) + p_{exp}(M,H)\right) - \left(p_{exp}(P,V) + p_{exp}(M,V)\right)$$

Input of P-polarization at θ

resolution of $\hat{S}_{PM}$ measurement

$$\varepsilon_r = \left(p_{exp}(P,H) + p_{exp}(P,V)\right) - \left(p_{exp}(M,H) + p_{exp}(M,V)\right)$$

Input of L-circular polarization at θ

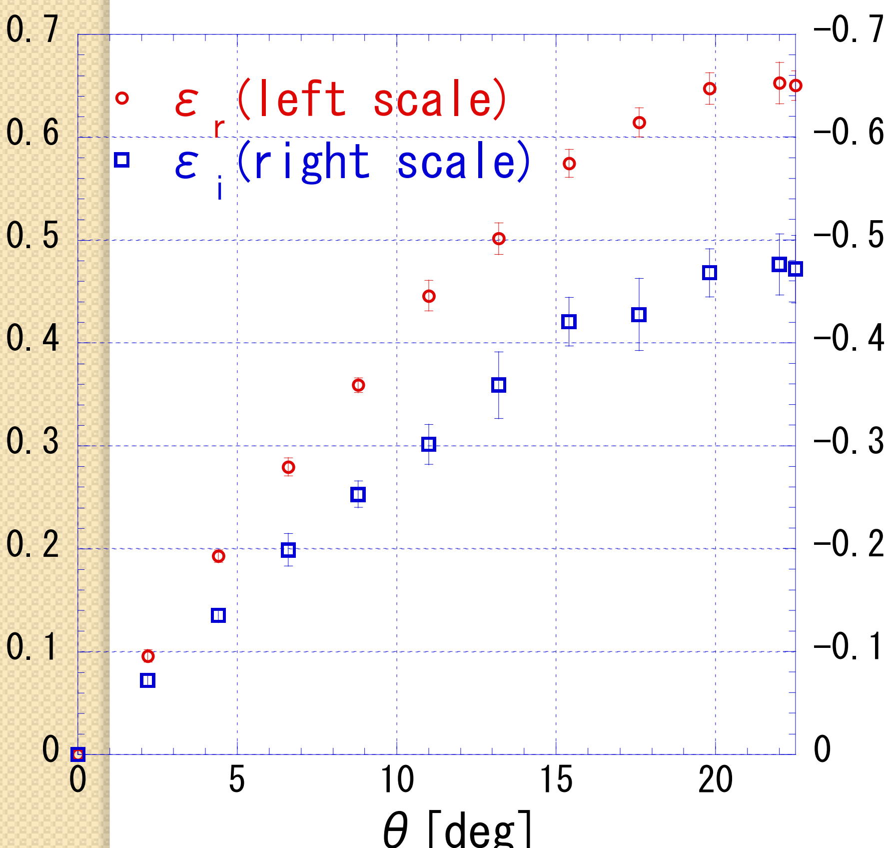
resolution of correlation between $\hat{S}_{HV}$ and $\hat{S}_{PM}$

$$\varepsilon_i = \left(p_{exp}(P,H) + p_{exp}(M,V)\right) - \left(p_{exp}(M,H) + p_{exp}(P,V)\right)$$

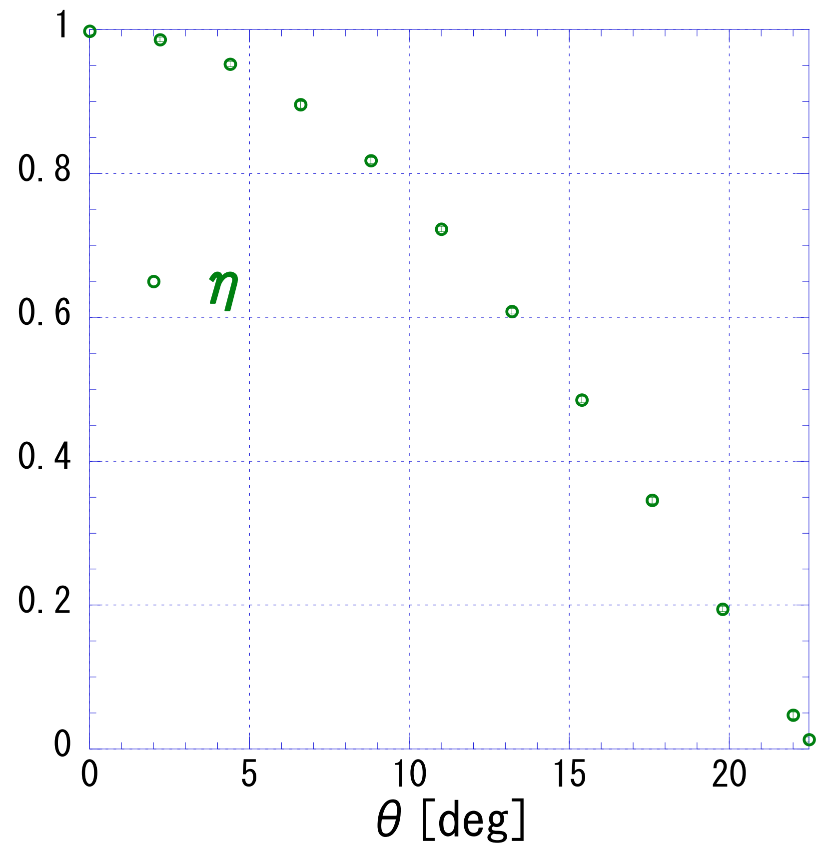
Error probability (at θ) $p(\pm s_{PM}, \pm s_{HV} \mid s_{PM}, s_{HV})$

Evaluation of resolution and back-action

initial : $|P\rangle$ initial : $|L\rangle$



initial : $|H\rangle$



Analysis (general case)

Error probability (at θ)

$$p(\, s_{PM}, s_{HV} \mid s_{PM}, s_{HV}) = \frac{1}{4}(1 + \eta + \varepsilon_r - i\varepsilon_i)$$

$$p(\, s_{PM}, -s_{HV} \mid s_{PM}, s_{HV}) = \frac{1}{4}(1 - \eta + \varepsilon_r + i\varepsilon_i)$$

$$p(\, -s_{PM}, s_{HV} \mid s_{PM}, s_{HV}) = \frac{1}{4}(1 + \eta - \varepsilon_r + i\varepsilon_i)$$

$$p(\, -s_{PM}, -s_{HV} \mid s_{PM}, s_{HV}) = \frac{1}{4}(1 - \eta - \varepsilon_r - i\varepsilon_i)$$

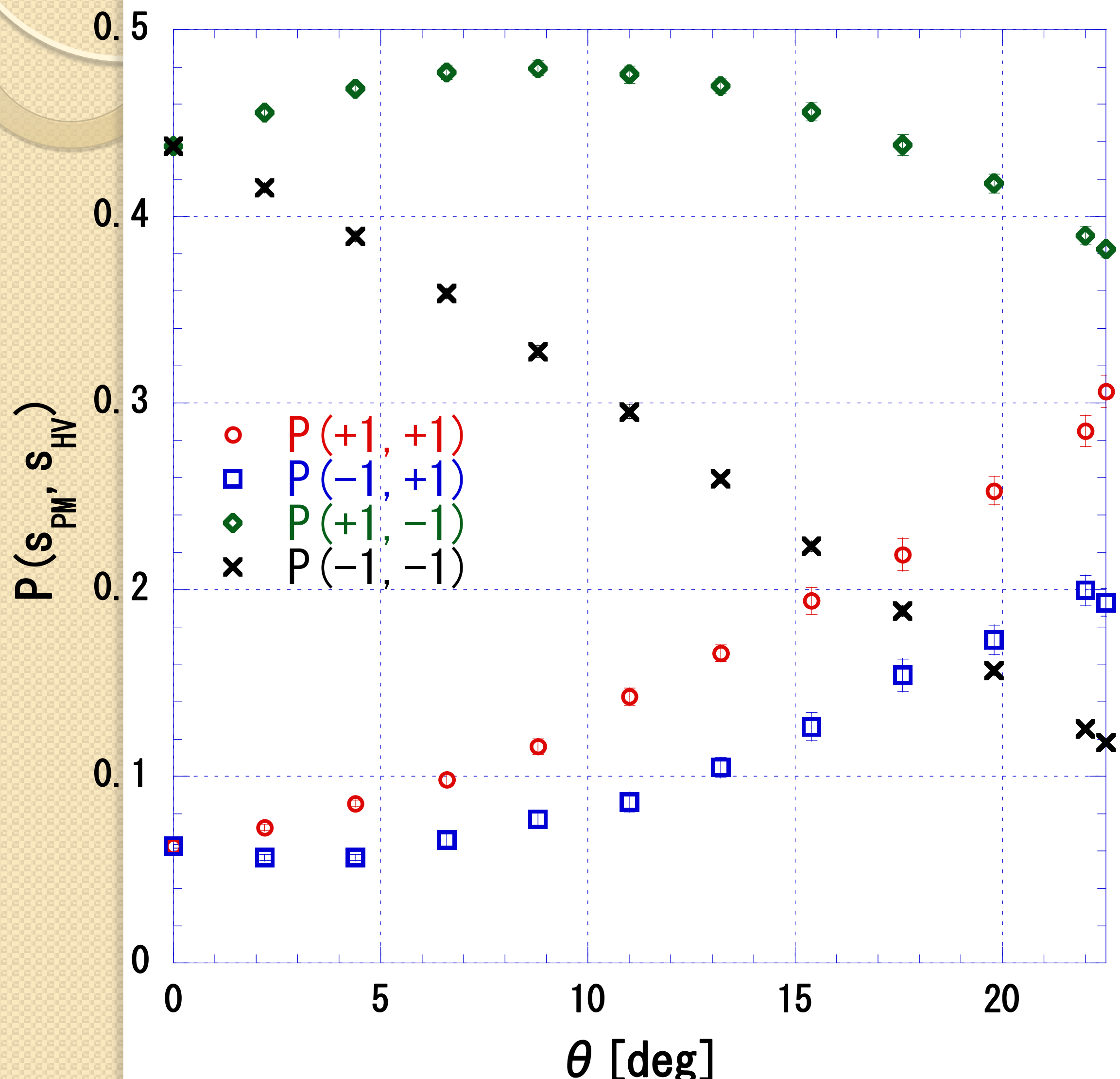
$$p_{exp}(s_{PM}, s_{HV}) = \underbrace{p(s_{PM}, s_{HV} | s_{PM}, s_{HV})}_{\text{Both no flips}} \rho_\psi(s_{PM}, s_{HV})$$

$$+ \underbrace{p(s_{PM}, s_{HV} | -s_{PM}, s_{HV})}_{\text{Only } s_{PM} \text{ flip}} \rho_\psi(-s_{PM}, s_{HV})$$

$$+ \underbrace{p(s_{PM}, s_{HV} | s_{PM}, -s_{HV})}_{\text{Only } s_{HV} \text{ flip}} \rho_\psi(s_{PM}, -s_{HV})$$

$$+ \underbrace{p(s_{PM}, s_{HV} | -s_{PM}, -s_{HV})}_{\text{Both flips}} \rho_\psi(-s_{PM}, -s_{HV})$$

Experimental joint probability



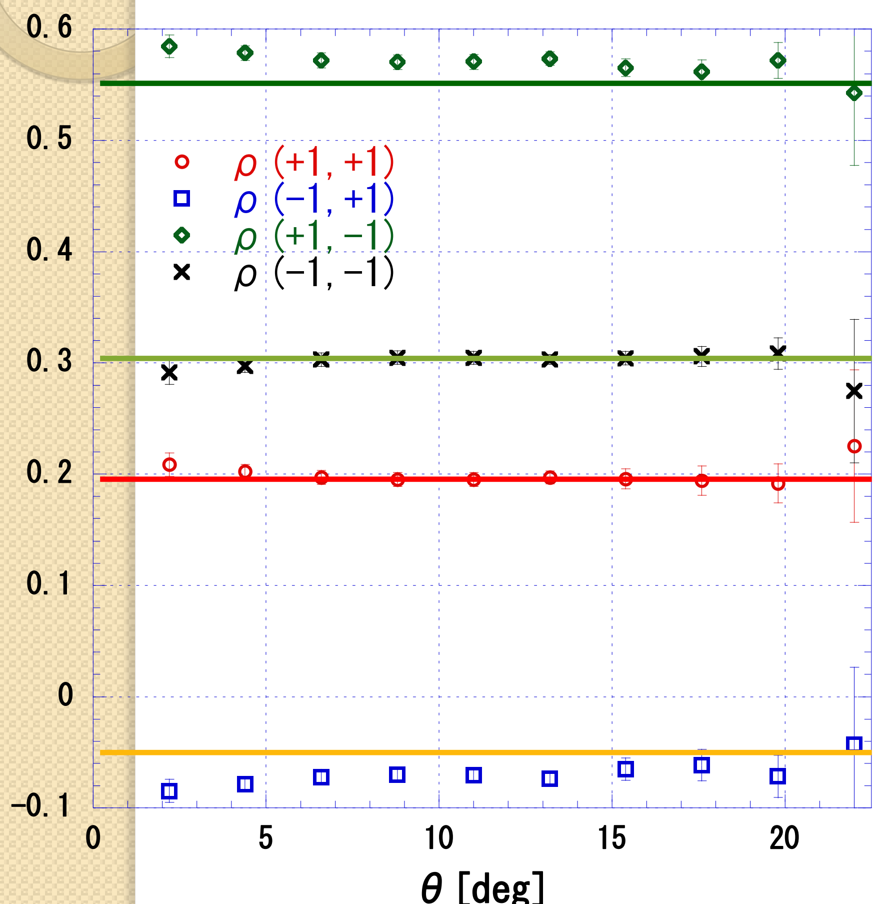
Initial state : elliptic

$$\begin{bmatrix} \cos\frac{3}{8}\pi \\ \sin\frac{3}{8}\pi \cdot e^{i\frac{\pi}{4}} \end{bmatrix}$$

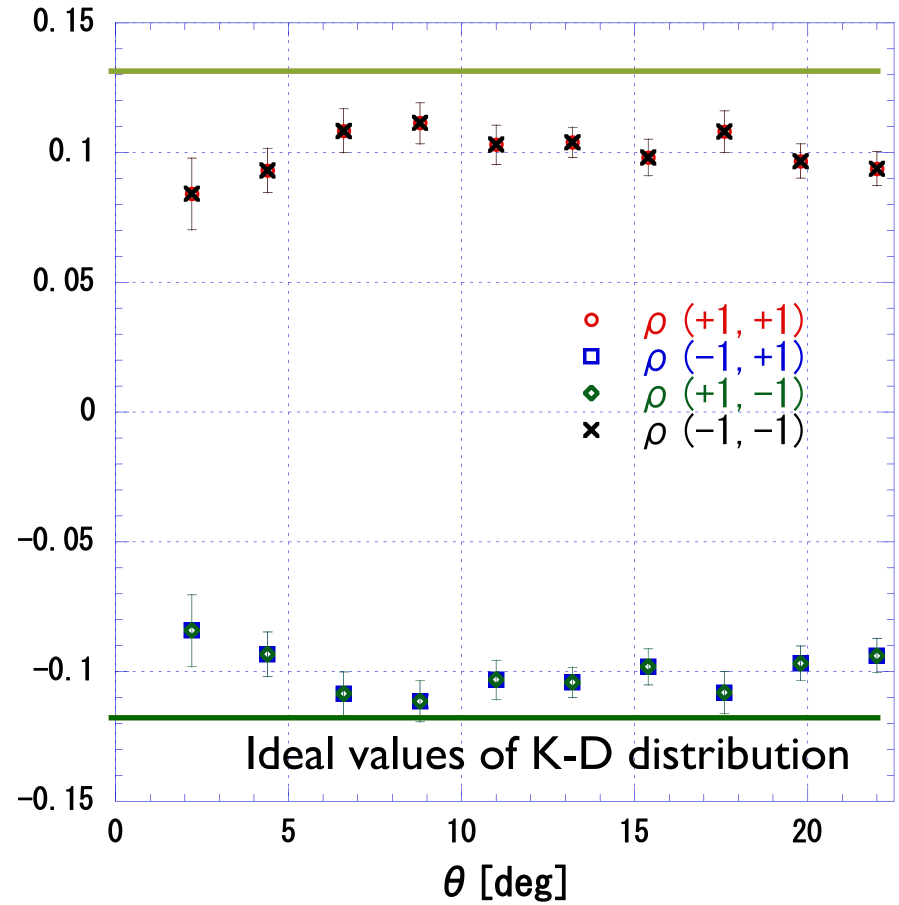
Strong dependence on θ

Reconstruction of complex probability

$Re[\rho(s_{PM}, s_{HV})]$



$Im[\rho(s_{PM}, s_{HV})]$



Independent in θ ➡ initial joint probability

4. Summary,

intriguing questions, prospective

Summary

- It is natural that quantum state is expressed by an negative or complex joint probability distribution.

- Experimentally-obtained probabilities are never identified to the probabilities before a measurement due to the interaction to the meter apparatus. Intrinsic probabilities are converted to positive probabilities by the measurement interaction.

- The results of weak measurement shows the intrinsic probability before the measurement process.

Intriguing questions

- Quantum tomography
  - Initial joint probability for any initial state (including a mixed state)
  - Comparison with a conventional tomography

- Measurement uncertainty (Ozawa formalism)
  - Detailed analysis of measurement process

- Possible extension to high dimension system
  - Analysis of back-action process

- Physical meaning of negative or complex probability
  - Quantum mechanics＝probability＋dynamics
  - Relation to unitary transformation
    （H. F. Hofmann, *New J. Phys*. 13, 103009 (2011) ）
  - Understanding of quantum mechanics by Quantum ergodicity
    （H. F. Hofmann, *Phy. Rev. A,* 89, 42115 (2014))

Prospective

My private opinion

Quantum-mechanical strange phenomena might be explained by K-D distribution.

- weak value and weak measurement
- understanding of measurement process
  - origin of measurement uncertainty
  - application to precise measurement
  - connection between quantum and classical pictures
- joint probability using entanglement state
  - Consistent connection between non-locality and locality
  - Reason of violation of Bell inequality
- Extension to higher dimensional system
  - analysis of back-action process
  - feasibility of complex joint probability

The analysis without the use of quantum theory is preferable