# A New Theory, Techniques and Alternatives for Direction-of-Arrival Estimation in Acoustic Signal Processing

#### Bandhit Suksiri

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1 Introduction: Preliminary Assessment, Objective, and New Approaches

- Research #1: Acoustic DOA Estimation by using Theory of Orthogonal Procrustes Analysis
- ③ Research #2: Extension Theory of Orthogonal Procrustes Analysis for Acoustic DOA Estimation
- ④ Research #3: Acoustic DOA and Variance Estimation via Complex-Valued Tensor Factorization

**5** Summary: Conclusions

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# Introduction: Direction-of-Arrival in the Field of Acoustics



Usages of acoustic Direction-of-Arrival (DOA) estimation

- ► Speech enhancement, source separation, human computer interaction
- Other applications; surveillance, automatic camera management

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### Introduction: Examples of the Applications



#### Reference:

Satoshi Tadokoro. (2017, December 25) Impulsing Paradigm Change through Disruptive Technologies Program (ImPACT): Tough Robotics Challenge. Retrieved from http://www.jst.go.jp/impact/en/program/07.html.



#### Reference:

Liu, Huawei; Li, Baoqing; Yuan, Xiaobing; Zhou, Qianwei; Huang, Jingchang. 2018. "A Robust Real Time Direction-of-Arrival Estimation Method for Sequential Movement Events of Vehicles." Sensors 18, no. 4: 992.

### Introduction: Preliminary Assessment

**Discussion**: The existing framework of acoustic DOA estimation



Problem: Lack of high efficient techniques and suitable theory when it comes to the acoustic DOA estimation

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### Introduction: Preliminary Assessment

**Discussion**: The existing framework of acoustic DOA estimation



Solution: Propose a new theory to extend the existing framework form wireless communication field to acoustic signal processing

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To propose an alternative theory for acoustic DOA estimation; this theory enables some useful techniques in wireless communication field to implement an acoustic DOA estimation for reducing a computational complexity and facilitating the estimation algorithm.

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- DOA via High-Order Generalized Singular Value Decomposition

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#### Research #1: System Overviews



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# Research #1: Microphone Structure



 $\delta = 2$ SNR  $\left(\frac{2\pi d}{\lambda}\right)^2$ , CRB represents the lowest error bound of DOA.



# Research #1: Microphone Structure





- Octagon CRB has only 5% smaller than L-Shaped CRB.
- DOA for L-Shaped array is widely proposed recently. (more DOA technique)

Y. Hua et al., "L-shaped for estimating 2D DOA," IEEE, 1991.

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### Research #1: General Model via Cross-Correlation



signals received by the microphone array

Signal Model at Z-axis :



signals received by the microphone array

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# Research #1: Well-known Direction-of-Arrival Methods



Well-known methods for estimating subarray angles:

- Multiple Signal Classification (MUSIC) [R.Schmidt, 1986]
- Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [R.Roy et al., 1989]
- 2-dimensional MUSIC [M.G.Porozantzidou et al., 2010]

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- > 2-dimensional MUSIC [M.G.Porozantzidou et al., 2010]
- These methods hold great promise in high efficient DOA method
- It is impossible to directly apply in human voice (multiple frequency)

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# Research #1: Angle Matrices on Wide Frequency Range



Sources always have various frequency ranges; music, human voice

# Research #1: Angle Matrices on Wide Frequency Range



- Sources always have various frequency ranges; music, human voice
- The left & right matrices are differences for all frequency bins
- However, they share same angles for all frequency bins



Signal subspace area represent the possibility of true DOAs



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- DOAs can be determined by intersecting the area between  $f_1$  and  $f_2$
- ► When there is no intersection area, it is impossible to detect DOAs



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- DOAs can be determined by intersecting the area between  $f_1$  and  $f_2$
- ▶ When there is no intersection area, it is impossible to detect DOAs
- Employing conventional DOA estimation in each frequency bins cannot obtain this intersection area

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Problem: How to get the center of intersection area (or signal subspace vector at f<sub>o</sub>) without employing the intersection area?



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#### ► Solution: Subspace transformation for all frequency bins

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### Research #1: Subspace Transformation Process



 $\blacktriangleright \mathbf{A}_{x}(\phi, f_{o}) = \mathbf{T}_{x\{f\}} \mathbf{A}_{x}(\phi, f), \quad \mathbf{A}_{z}(\theta, f_{o}) = \mathbf{T}_{z\{f\}} \mathbf{A}_{z}(\theta, f), \quad \forall f.$ 

## Research #1: Subspace Transformation Process



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Optimization Problem for Estimating  $T_{x\{f\}}$ ,  $T_{z\{f\}}$  (impracticable!)

$$\underset{\boldsymbol{T}_{x\{f\}}}{\operatorname{ninimize}} \quad \left\| \boldsymbol{A}_{x}\left(\phi, f_{\mathsf{o}}\right) - \boldsymbol{T}_{x\{f\}} \boldsymbol{A}_{x}\left(\phi, f\right) \right\|_{\mathsf{F}}^{2},$$

$$\begin{array}{l} \underset{\boldsymbol{\mathcal{T}}_{z\{f\}}}{\text{minimize}} \quad \left\|\boldsymbol{A}_{z}\left(\boldsymbol{\theta},f_{\mathsf{o}}\right)-\boldsymbol{\mathcal{T}}_{z\{f\}}\boldsymbol{A}_{z}\left(\boldsymbol{\theta},f\right)\right\|_{\mathsf{F}}^{2} \end{array}$$

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# Research #1: Possible Solution for Estimating $T_{x\{f\}}$



• Matrices above are valid iff x(t, f), z(t, f) are not a stationary signal

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#### Practical Optimization Problem for Estimating $T_{x\{f\}}$

$$\begin{array}{l} \underset{\boldsymbol{T}_{x\{f\}}}{\text{minimize}} & \left\| \boldsymbol{R}_{xz\{f_{0},f_{0}\}} - \boldsymbol{T}_{x\{f\}} \boldsymbol{R}_{xz\{f,f_{0}\}} \right\|_{\mathsf{F}}^{2} ,\\ \\ \underset{\boldsymbol{T}_{x\{f\}}}{\text{minimize}} & \left\| \boldsymbol{R}_{xx\{f_{0},f_{0}\}} - \boldsymbol{T}_{x\{f\}} \boldsymbol{R}_{xx\{f,f_{0}\}} \right\|_{\mathsf{F}}^{2} . \end{array}$$

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# Research #1: Solution for Estimating $T_{x\{f\}}, T_{z\{f\}}$

Given a temporal frequency f, the solution matrices are defined as:

$$\begin{split} \Psi_{x\{f\}} &= \textit{\textbf{R}}_{xz\{f,f_{o}\}}\textit{\textbf{R}}_{xz\{f_{o},f_{o}\}}^{\mathsf{H}}, \\ \Psi_{z\{f\}} &= \textit{\textbf{R}}_{zx\{f,f_{o}\}}\textit{\textbf{R}}_{zx\{f_{o},f_{o}\}}^{\mathsf{H}}, \end{split} \qquad \text{or} \quad \end{split}$$

$$\begin{split} \Psi_{x\{f\}} &= R_{xx\{f,f_{o}\}} R_{xx\{f_{o},f_{o}\}}^{\mathsf{H}}, \\ \Psi_{z\{f\}} &= R_{zz\{f,f_{o}\}} R_{zz\{f_{o},f_{o}\}}^{\mathsf{H}}. \end{split}$$

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Given a temporal frequency f, the solution matrices are defined as:

$$\begin{split} \Psi_{x\{f\}} &= R_{xz\{f,f_{0}\}} R^{H}_{xz\{f_{0},f_{0}\}}, & \Psi_{x\{f\}} &= R_{xx\{f,f_{0}\}} R^{H}_{xx\{f_{0},f_{0}\}}, \\ \Psi_{z\{f\}} &= R_{zx\{f,f_{0}\}} R^{H}_{zx\{f_{0},f_{0}\}}, & \text{or} & \Psi_{z\{f\}} &= R_{zz\{f,f_{0}\}} R^{H}_{zz\{f_{0},f_{0}\}}. \end{split}$$

#### Theorem: Orthogonal Procrustes Analysis

The one possible solution of the optimization problems are given as:

$$\boldsymbol{T}_{x\{f\}} = \boldsymbol{V}_{x_s\{f\}} \boldsymbol{U}_{x_s\{f\}}^{\mathsf{H}},$$

$$\boldsymbol{T}_{z\{f\}} = \boldsymbol{V}_{z_s\{f\}} \boldsymbol{U}_{z_s\{f\}}^{\mathsf{H}},$$

where  $\pmb{\Psi}_{x\{f\}}$ ,  $\pmb{\Psi}_{z\{f\}}$  are factorized via Singular-Value Decomposition;

$$\begin{split} \Psi_{x\{f\}} &= U_{x_{s}\{f\}} \Sigma_{x_{s}\{f\}} V_{x_{s}\{f\}}^{\mathsf{H}} + U_{x_{w}\{f\}} \Sigma_{x_{w}\{f\}} V_{x_{w}\{f\}}^{\mathsf{H}}, \\ \Psi_{z\{f\}} &= U_{z_{s}\{f\}} \Sigma_{z_{s}\{f\}} V_{z_{s}\{f\}}^{\mathsf{H}} + U_{z_{w}\{f\}} \Sigma_{z_{w}\{f\}} V_{z_{w}\{f\}}^{\mathsf{H}}. \end{split}$$

# Research #1: Solution for Estimating $T_{x\{f\}}$ , $T_{z\{f\}}$



The above components can be referred to as singular values

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# Research #1: New Framework for Estimating DOA



- Employ SVD in each frequency bins
- ▶ Transform all *f* into *f*<sub>o</sub> space
- *R<sub>xz</sub>* is now able to perform SVD; it is compatible with recent subspace methods.

# Research #1: New Framework for Estimating DOA



- Employ SVD in each frequency bins
- Transform all f into f<sub>o</sub> space
- *R<sub>xz</sub>* is now able to perform SVD; it is compatible with recent subspace methods.

#### DOA Estimation Steps:

- 1. Obtain  $\boldsymbol{T}_{x\{f\}}, \boldsymbol{T}_{z\{f\}}$
- 2. Calculate  $R_{xz}$
- 3. Performing SVD of  $R_{xz} = U\Sigma V^{H}$
- 4. Estimate DOA angles  $\boldsymbol{U} \equiv \boldsymbol{A}_{x} (\phi, f_{o})$   $\boldsymbol{V} \equiv \boldsymbol{A}_{z} (\theta, f_{o})$ (X. Nie *et al.*, 2015)
  - Possible to change another method



X. Nie *et al.*, "Array aperture extension algorithm for 2-d doa estimation with l-shaped array," 2015.

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## Research #1: Experimentation

Experiment Setup :



- Number of Microphone : 8
- Capture time : 5 second
- Reverberation time : 0.3 second
- SNR value : 22.78 dB
- Focus only front direction



- KUT meeting room (A511)
- 2 3 People speak to the microphones simultaneity

### Research #1: Experimental Result

- Performance under the real environment
- Azimuth ( $\psi$ ) and elevation (or zenith,  $\theta$ ) angles are considered
- $\bullet \ \cos \phi_k = \sin \theta_k \cos \psi_k$

	Incident S	ources	Average of Calculated DOAs (Degree)					
Number	Position	Angle (Degree)	IMUSIC	TOFS	CSS-PGAM	Proposed Method		
2	$\psi_1$	45.000	58.292	57.231	60.276	57.877		
	$\theta_1$	50.000	56.207	55.119	53.537	50.140		
	$\psi_2$	135.000	139.610	142.596	failed	152.335		
	$\theta_2$	100.000	91.673	91.788	failed	99.245		
3	$\psi_1$	45.000	failed	failed	57.489	49.943		
	$ heta_1$	55.000	failed	failed	57.079	48.291		
	$\psi_2$	100.000	failed	failed	97.312	98.575		
	$\theta_2$	95.000	failed	failed	93.189	93.505		
	$\psi_3$	135.000	140.857	141.631	failed	149.198		
	$\theta_3$	135.000	93.449	94.939	failed	119.271		



- [IMUSIC] G. Su *et al.*, "The signal subspace approach for multiple wide-band emitter location," 1983.
- [TOFS] H. Yu *et al.*, "A new method for wideband doa estimation," 2007.
  - [CSS-PGAM] B. Suksiri et al., "A computationally efficient wideband direction-of-arrival estimation method for l-shaped microphone arrays," 2018.

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**Discussion**: Why Did This Happen?

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- Answer: Because  $T_{x\{f_1\}}, T_{x\{f_2\}}, \cdots$  are estimated separately

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▶ Solution:  $T_{x\{f_1\}}, T_{x\{f_2\}}, \cdots$  have to be estimated simultaneously



where  $\sum_{k=1}^{K} \sigma_k^2(\mathbf{A})$  is the sum-of-squares K largest singular values of  $\mathbf{A}$ .

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#### Proposed Theorem: Extension Theory of Orthogonal Procrustes

The one possible solution of the optimization problems are given as:

$$\boldsymbol{T}_{x\{f\}} = \boldsymbol{V}_{\boldsymbol{e}_{\{f\}_{s}}} \boldsymbol{U}_{\boldsymbol{e}_{\{f\}_{s}}}^{\dagger},$$

where  $E_{x\{f\}}$  is factorized via Generalized Singular-Value Decomposition;

$$\boldsymbol{E}_{x\{f\}} = \boldsymbol{U}_{e_{\{f\}_{s}}} \boldsymbol{\Sigma}_{e_{\{f\}_{s}}} \boldsymbol{V}_{e_{\{f\}_{s}}}^{\mathsf{H}} + \boldsymbol{U}_{e_{\{f\}_{n}}} \boldsymbol{\Sigma}_{e_{\{f\}_{n}}} \boldsymbol{V}_{e_{\{f\}_{n}}}^{\mathsf{H}}$$

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$$\begin{aligned} \boldsymbol{E}_{x\{f\}} &= \boldsymbol{U}_{e_{\{f\}_{s}}}\boldsymbol{\Sigma}_{e_{\{f\}_{s}}}\boldsymbol{V}_{e_{\{f\}_{s}}}^{\mathsf{H}} + \boldsymbol{U}_{e_{\{f\}_{n}}}\boldsymbol{\Sigma}_{e_{\{f\}_{n}}}\boldsymbol{V}_{e_{\{f\}_{n}}}^{\mathsf{H}}. \end{aligned}$$
Note that  $\begin{bmatrix} \boldsymbol{U}_{e_{\{f\}_{s}}} & \boldsymbol{U}_{e_{\{f\}_{n}}} \end{bmatrix}$  is not unitary,  $\boldsymbol{V}_{e_{\{f\}_{s}}}$  does not depend on  $f$ ;  
 $\boldsymbol{V}_{e_{s}} &= \boldsymbol{V}_{e_{\{f_{1}\}_{s}}} = \boldsymbol{V}_{e_{\{f_{2}\}_{s}}} = \cdots = \boldsymbol{V}_{e_{\{f_{P}\}_{s}}}. \end{aligned}$ 

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▶ Question: How to factorize it simultaneously (not separately)?

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Question: How to factorize it simultaneously (not separately)?
 Answer: High-Order Generalized Singular-Value Decomposition

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### Definition of High-Order Generalized Singular-Value Decomposition

$$\begin{bmatrix} \boldsymbol{E}_{x\{f_1\}} \\ \boldsymbol{E}_{x\{f_2\}} \\ \vdots \\ \boldsymbol{E}_{x\{f_P\}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{U}_{e\{f_1\}_s} \boldsymbol{\Sigma}_{e\{f_1\}_s} \\ \boldsymbol{U}_{e\{f_2\}_s} \boldsymbol{\Sigma}_{e\{f_2\}_s} \\ \vdots \\ \boldsymbol{U}_{e\{f_P\}_s} \boldsymbol{\Sigma}_{e\{f_P\}_s} \end{bmatrix} \boldsymbol{V}_{e_s}^{\mathsf{H}} + \begin{bmatrix} \boldsymbol{U}_{e\{f_1\}_n} \boldsymbol{\Sigma}_{e\{f_1\}_n} \\ \boldsymbol{U}_{e\{f_2\}_n} \boldsymbol{\Sigma}_{e\{f_2\}_n} \\ \vdots \\ \boldsymbol{U}_{e\{f_P\}_n} \boldsymbol{\Sigma}_{e\{f_P\}_n} \end{bmatrix} \boldsymbol{V}_{e_n}^{\mathsf{H}}.$$

### Definition of High-Order Generalized Singular-Value Decomposition

$$\begin{bmatrix} \boldsymbol{E}_{x\{f_1\}} \\ \boldsymbol{E}_{x\{f_2\}} \\ \vdots \\ \boldsymbol{E}_{x\{f_P\}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{U}_{e\{f_1\}_s} \boldsymbol{\Sigma}_{e\{f_1\}_s} \\ \boldsymbol{U}_{e\{f_2\}_s} \boldsymbol{\Sigma}_{e\{f_2\}_s} \\ \vdots \\ \boldsymbol{U}_{e\{f_P\}_s} \boldsymbol{\Sigma}_{e\{f_P\}_s} \end{bmatrix} \boldsymbol{V}_{e_s}^{\mathsf{H}} + \begin{bmatrix} \boldsymbol{U}_{e\{f_1\}_n} \boldsymbol{\Sigma}_{e\{f_1\}_n} \\ \boldsymbol{U}_{e\{f_2\}_n} \boldsymbol{\Sigma}_{e\{f_2\}_n} \\ \vdots \\ \boldsymbol{U}_{e\{f_P\}_n} \boldsymbol{\Sigma}_{e\{f_P\}_n} \end{bmatrix} \boldsymbol{V}_{e_n}^{\mathsf{H}}.$$

Important Criteria: 
$$V_{e_s} = V_{e_{\{f_1\}_s}} = V_{e_{\{f_2\}_s}} = \dots = V_{e_{\{f_P\}_s}}$$

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Important Criteria: 
$$\boldsymbol{V}_{e_s} = \boldsymbol{V}_{e_{\{f_1\}_s}} = \boldsymbol{V}_{e_{\{f_2\}_s}} = \cdots = \boldsymbol{V}_{e_{\{f_P\}_s}}$$
 (valid!)

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Important Criteria: 
$$V_{e_s} = V_{e_{\{f_1\}_s}} = V_{e_{\{f_2\}_s}} = \dots = V_{e_{\{f_P\}_s}}$$
 (valid!)  
New Solution of  $T_{x\{f\}}$ : 
$$\begin{bmatrix} T_{x\{f_1\}} \\ T_{x\{f_2\}} \\ \vdots \\ T_{x\{f_P\}} \end{bmatrix} = V_{e_s} \begin{bmatrix} U_{e_{\{f_1\}_s}} \\ U_{e_{\{f_2\}_s}}^{\dagger} \\ \vdots \\ U_{e_{\{f_P\}_s}}^{\dagger} \end{bmatrix}$$



S.P. Ponnapalli *et al.*, "Higher-Order Generalized Singular Value Decomposition for Comparison of Global mRNA Expression from Multi-organisms," PLOS ONE, 2011.

## Research #2: Performance Evaluation

- Performance under the reverberation environment (RT60)
- $(\theta_k^{\text{DOA}}, \phi_k^{\text{DOA}})$  are placed at (41.41°, 60°), (60°, 45°), (75.52°, 30°)

## Research #2: Performance Evaluation

- Performance under the reverberation environment (RT60)
- $(\theta_k^{\text{DOA}}, \phi_k^{\text{DOA}})$  are placed at (41.41°, 60°), (60°, 45°), (75.52°, 30°)



## Research #2: Experimentation

Experiment Setup :



- Number of Microphone : 8
- Capture time : 5 second
- Reverberation time : 0.3 second
- SNR value : 22.78 dB
- Focus only front direction



- KUT meeting room (A511)
- 2 3 People speak to the microphones simultaneity

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### Research #2: Experimental Result

Performance under the real environment

## Research #2: Experimental Result

#### Performance under the real environment

Incident Sources			RMSE of DOAs (Degree)						
Number	Position	Angle (Degree)	IMUSIC	TOFS	TOPS	Squared TOPS	Proposed Method with MUSIC	Proposed Method with ESPRIT	
1	$\phi_1$	96	0.3050	0.2050	1.0950	1.3350	0.7750	0.7074	
	$\theta_1$	86	0.5400	1.2600	1.2750	2.0150	0.5700	0.6915	
		Average	0.4225	0.7325	1.1850	1.6750	0.6725	0.6995	
2	$\phi_1$	65	1.1857	1.7286	20.0143	28.5857	1.5000	2.0284	
	$\theta_1$	150	9.6000	6.6857	26.3571	39.7857	8.8143	8.6800	
	$\phi_2$	55	1.0714	1.6857	22.2571	19.4000	2.9714	3.8695	
	$\theta_2$	100	8.3714	8.3857	5.0143	6.7857	6.6714	3.1630	
		Average	5.0571	4.6214	18.4107	23.6393	4.9893	4.4353	
3	$\phi_1$	58	2.1400	2.3900	46.5500	52.8100	3.6600	4.0334	
	$\theta_1$	55	55.0000	55.0000	55.0000	55.0000	9.4300	4.1057	
	$\phi_2$	100	1.8400	2.0000	41.5700	62.4000	1.8700	2.4554	
	$\theta_2$	95	95.0000	83.4200	52.4500	71.4800	9.7700	5.8638	
	$\phi_3$	130	10.9300	11.8900	28.8300	32.2800	8.2500	6.9071	
	$\theta_3$	120	26.9800	25.8400	16.1200	18.0100	5.9400	7.3165	
		Average	31.9817	30.0900	40.0867	48.6633	6.4867	5.1137	

- [IMUSIC] G. Su et al., "The signal subspace approach for multiple wide-band emitter location," 1983.
  - [TOPS] Y. S. Yoon *et al.*, "Tops: new doa estimator for wideband signals," 2006.
- [TOFS] H. Yu et al., "A new method for wideband doa estimation," 2007.
- [Squared-TOPS] K. Okane *et al.*, "Resolution improvement of wideband direction-of-arrival estimation "squared-tops"," 2010.
- - [MUSIC] R. Schmidt, "Multiple emitter location and signal parameter estimation," 1986.



[ESPRIT] R. Roy *et al.*, "ESPRIT-estimation of signal parameters via rotational invariance techniques," 1989.

1 Introduction: Preliminary Assessment, Objective, and New Approaches

- 2 Research #1: Acoustic DOA Estimation by using Theory of Orthogonal Procrustes Analysis
- ③ Research #2: Extension Theory of Orthogonal Procrustes Analysis for Acoustic DOA Estimation
- ④ Research #3: Acoustic DOA and Variance Estimation via Complex-Valued Tensor Factorization

**5** Summary: Conclusions

- In research #1, accuracy performance of acoustic DOA estimation is improved, but information of frequency bins are compressed and lost
- ► In research #2, the accuracy performance is further improved, but we didn't address how to obtain information of frequency bins

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Given an 3<sup>rd</sup>-order tensor <u>Q</u> ∈ C<sup>M×F×M</sup> and the inner index K
 D<sub>xz{f}</sub> are rearranged by lateral slices of <u>Q</u>



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#### Tensor Representation

$$\begin{split} \underline{\boldsymbol{Q}} &= \left[ \left[ \boldsymbol{A}_{x} \left( \phi, f_{o} \right), \boldsymbol{P}, \bar{\boldsymbol{A}}_{z} \left( \theta, f_{o} \right) \right] \right], \\ \underline{\boldsymbol{Q}}_{:,f,:} &= \boldsymbol{D}_{xz\{f\}}, (\text{lateral slice represent.}) \\ \boldsymbol{D}_{xz\{f\}} &= \boldsymbol{T}_{x\{f\}} \boldsymbol{R}_{xz\{f,f\}} \boldsymbol{T}_{z\{f\}}^{\mathsf{H}}. \end{split}$$

*R*<sub>xz{f,f}</sub> ∈ C<sup>M×M</sup> is the sample cross-correlation matrix
 *T*<sub>x{f}</sub>, *T*<sub>z{f}</sub> ∈ C<sup>M×M</sup> are the transformation matrices
 *A*<sub>x</sub> (φ, f), *A*<sub>z</sub> (θ, f) ∈ C<sup>M×K</sup> is the array manifold matrices
 *Ā*<sub>z</sub> (θ, f<sub>0</sub>) ∈ C<sup>M×K</sup> is complex conjugate of the elements of *A*<sub>z</sub> (θ, f<sub>0</sub>)
 *P* ∈ ℝ<sup>F×K</sup><sub>>0</sub> is the sample variance matrix for all frequency

## Research #3: DOA Estimation via Tensor Factorization

- ▶ To isolate  $A_x$ , P,  $A_z$  from  $\underline{Q}$ , tensor factorization is employed
- Employing Complex-valued Parallel Factor Analysis model; <sup>1</sup>

$$\boldsymbol{P} = \begin{pmatrix} \sigma_{s_{1}\{f_{\min}\}}^{2} & \sigma_{s_{2}\{f_{\min}\}}^{2} & \cdots & \sigma_{s_{K}\{f_{\min}\}}^{2} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{s_{1}\{f\}}^{2} & \sigma_{s_{2}\{f\}}^{2} & \cdots & \sigma_{s_{K}\{f\}}^{2} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{s_{1}\{f_{0}\}}^{2} & \sigma_{s_{2}\{f_{0}\}}^{2} & \cdots & \sigma_{s_{K}\{f_{0}\}}^{2} \end{pmatrix}$$

•  $\sigma_{s_k\{f\}}^2$  is a variance of the signal source  $s_k(t, f)$  or singular values

► Estimate DOA angles <sup>2</sup> :  $\boldsymbol{U} \equiv \boldsymbol{A}_{x} \left( \phi, \boldsymbol{f_{o}} \right), \ \boldsymbol{V} \equiv \boldsymbol{A}_{z} \left( \theta, \boldsymbol{f_{o}} \right)$ 

<sup>1</sup>N.D.Sidiropoulos *et al.*, "Blind PARAFAC Receivers for DS-CDMA System," 2000.



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### Research #3: Variance Estimation Performance

The three sources were the following piano notes; (a, d) G5 - 783.99 Hz, (b, e) C5 or Tenor C (523.25 Hz), (c, f) A4 or A440 (440 Hz).
## Research #3: Variance Estimation Performance

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## Research #3: Angle Estimation Performance

 $(\theta_k^{\text{DOA}}, \phi_k^{\text{DOA}})$  are placed at (41.41°, 60°), (60°, 45°), and (75.53°, 30°)

 $\left(\theta_k^{\rm DOA},\phi_k^{\rm DOA}\right)$  are placed at (41.41°,60°), (60°,45°), and (75.53°,30°)



Number of microphone each subarray on (a) = 6, (b) = 8, and (c) = 10

- - [IMUSIC] G.Su *et al.*, "The signal subspace approach for multiple wide-band emitter location," 1983.
  - [TOPS] Y.S.Yoon *et al.*, "Tops: new doa estimator for wideband signals," 2006.
  - [TOFS] H.Yu *et al.*, "A new method for wideband doa estimation," 2007.
- [WS-TOPS] H.Hirotaka et al., "Doa estimation for wideband signals based on weighted squared tops," 2016.
- [CSS-DOP] B.Suksiri *et al.*, "A Highly Efficient Wideband Two-Dimensional Direction Estimation Method with L-Shaped Microphone Array," IEICE-EA, 2019.

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**5** Summary: Conclusions

#### Conclusions:

 An extension of techniques, new framework and suitable theory for estimating acoustic DOAs are presented.

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### Conclusions:

- An extension of techniques, new framework and suitable theory for estimating acoustic DOAs are presented.
- These alternative provide a new framework for recent narrowband subspace methods to estimating acoustic DOA; for reducing the computational complexity and facilitating the algorithm.

### Contribution:

This work bridge a research gap of acoustic source compatibility on the recent narrowband and wideband subspace methods to estimate DOA of the acoustic sources directly and effectively.

# Summary: Publications

- Authors: Bandhit Suksiri, Masahiro Fukumoto
- Journal papers:
  - #1. Submitted to: J-STAGE Journal of Signal Processing
    Tittle: Multiple Frequency and Source Angle Estimation by Gaussian Mixture Model with Modified Microphone Array Data Model
     Status: published on July 20, 2017
  - #2. Submitted to: IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences (IEICE-EA) Tittle: A Highly Efficient Wideband Two-Dimensional Direction Estimation Method with L-Shaped Microphone Array Status: accepted on July 31, 2019, publication in November 2019 Rank: Q4 (JCR 2018)
  - #3. Submitted to: Sensors

**Tittle:** An Efficient Framework for Estimating Direction of Multiple Sound Sources using Higher-Order Generalized Singular Value Decomposition

Status: published on July 5, 2019 Rank: Q1 (JCR 2018)

# Summary: Publications

### Peer-reviewed international conference:

- #1. Conference: 14th RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing (RISP NCSP'17)
   Tittle: Multiple Frequency and Source Angle Estimation by Gaussian Mixture Model with Modified Microphone Array Data Model
   Award: Student Paper Award
- #2. Conference: 9th Asia-Pacific Signal and Information Processing Association Annual Summit and Conference (APSIPA ASC 2017)
   Tittle: Enhanced Array Manifold Matrices for L-Shaped Microphone Array-based 2-D DOA Estimation
- #3. Conference: 50th IEEE International Symposium on Circuits and Systems (ISCAS 2018)
   Tittle: A Computationally Efficient Wideband Direction-of-Arrival Estimation Method for L-Shaped Microphone Arrays
   Award: IEEE CAS Student Travel Grant Award

#4. Conference: 16th International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology (ECTI-CON 2019) Tittle: Wideband Direction-of-Arrival Estimation with Cross-Sample

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#### **Domestic conference:**

#1. **Conference:** 31st Signal Processing Symposium (31st SIP SYMPOSIUM) **Tittle:** Wavelet Analysis for Multiple Frequency and Signal Classification in Linear Phased Array Model (presenting in English) #2. **Conference:** 32nd Signal Processing Symposium (32nd SIP SYMPOSIUM) Tittle: A Novel L-Shaped Microphone Array-based Wideband Direction of Arrival Estimation Method using the Special Cross-correlation Matrix (presenting in English) #3. **Conference:** 33rd Signal Processing Symposium (33rd SIP SYMPOSIUM) Tittle: Complex-valued Tensor Factorization on Wideband Two-dimensional Direction Estimation Method with L-Shaped Microphone Array (presenting in Japanese)