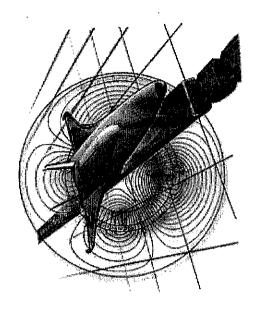


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# 2 DOF Magnetically Suspended Manipulation with Permanent Magnet Motion Control

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#### Abstract

This paper describes a control method of 2 DOF magnetically suspended manipulation system. The suspension mechanism is that suspension force is controlled by air gap adjusting. The feature of this mechanism is use of permanent magnets and linear actuators. We study the stability of a 2 DOF suspension system which manipulate the object in the vertical plane. To analyze the stability of the system, we assume that the attractive force acts on the direction from the magnet tip to the center of the object, and is inversely proportional to the square of the air gap length. First, the principle of the suspension mechanism is explained and a prototype 2 DOF system is introduced. We make a linearized model of the prototype system. The feedback gains are calculated by linear control theory. We consider two deferent system, one is a permanent magnet position input system and the other is the system which inputs are driving forces of the magnets. We carry out the numerical simulations on these nonlinear 1 DOF and 2 DOF systems. As the results, on the force input system, the feedback gains of 2 DOF are deferent from 1 DOF system, and the stable range of 2 DOF system is very narrow.

#### Introduction

Magnetic suspension system which controls the attractive force by adjusting the air gap has been developed [1]. The feature of this suspension mechanism is use of permanent magnets and linear actuators. To control supporting forces, system adjusts the air gap length by permanent magnet movements. There were some 1 DOF developments of this suspension system[2]-[4].

The suspension mechanism uses a permanent magnet not a electromagnet, so there is no heat generation, no need of volume for a coil, and no actuator installation near the object by using telecontrol of permanent magnet movement. So, this suspension mechanism may be suitable for micro manipulation. When this suspension mechanism is used for the operation of manipulation, the performance of a 1 DOF system is insufficient on the points of the stability of passive control direction, countermeasures for various shape of the suspended object, controllable area, and so on, We need multi-DOF suspension systems. Multi-DOF suspension systems can not be considered that the system consists of some individual subsystems along magnet movement directions. Because the suspension system does not make the magnetic path to be closed, the air gap length is large compared with normal magnetic bearings.

In this paper, we study the 2 DOF suspension system which manipulate the object in the vertical plane, as the first step of multi-DOF micro manipulation. Stability of a 2 DOF magnetic suspension system is studied compared with a 1 DOF system. First, the principle of suspension system is explained and a 2 DOF system is introduced, analyzed, and modeled. There are two models of suspension system. One is position input system, and the other is actuator force input system. The feedback gains are calculated by linear control theory and numerical simulations are carried out comparing with 1 DOF system.

#### Principle of Suspension System

A suspension system with a permanent magnet and linear actuator is proposed as shown in Fig 1 [1]. A ferromagnetic body is suspended by the attractive force from a permanent magnet positioned above. The magnet is driven by an actuator. The direction of levitation is vertical, and the magnet and the object move only in this direction. The equilibrium position determined by a balance

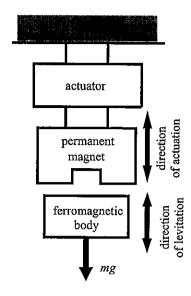


Fig. 1: Outline of Magnetic Suspension System

between the gravity force and the magnet force.

If the actuator does not actively control the magnet's position, the levitated object will either fall or adhere to the magnet. However servo-control of the actuator can make this system stable. Because there is a smaller attractive force for a larger air gap between the permanent magnet and object, the actuator drives the magnet upwards in response to object movement from its equilibrium position towards the magnet. Similarly, the actuator drives the magnet downwards in response to object movement away from the magnet. In this way, the object can be stably suspended without contact. In comparison to the electrical control method of electromagnetic suspension systems, this system is a mechanical control magley system.

#### 2 DOF Suspension System

A photograph of a prototype 2 DOF suspension system is shown in Fig. 2. There are two voice coil actuators which drive permanent magnets. The actuators are installed to hemicycle rail and the directions of the magnet movements can be adjusted by the installed position. The suspended object is an iron ball, and is manipulated by the movements of the two magnets. The movements of the magnets and the iron ball are sensed by the gap sensors and the photo sensor, respectively.

The outlines of a 2 DOF suspension system and a 1 DOF suspension system are shown in Figure 3. In the 2 DOF suspension system, O is the origin of the coordinate frame and the circle indicated by (x, y) is the suspended object. The suspended object is acted on by the force of gravity in the vertical direction. The rectangles on the X and Y axes are permanent magnets, and each magnet can move

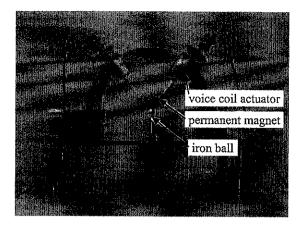


Fig. 2: Photograph of Prototype Suspension System

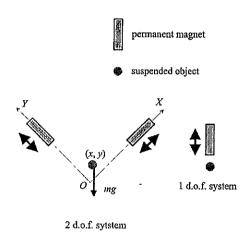


Fig. 3: Outline of Two Types of Suspension System

along its own axis respectively. These two axis are perpendicular to each other and the gradients of two axes are both  $\pi/4$  in the following analysis.

The attractive forces of permanent magnets acting on the object has the direction from the center of the object to the near tip of the magnet and the strength which is inverse proportion to the length between the object and the tip. The size of the object can be neglected. The potential forces (lateral force) of the magnets are assumed to be same as the lateral component of the attractive force vectors.

In the 1 DOF suspension system, the magnet force and the gravity force are balanced in the equilibrium, and the permanent magnet is driven in the vertical direction to make system.stable. The stability of the horizontal direction is controlled passively. We consider the movement of vertical direction in the 1 DOF system.

## Model of Suspension System

To analyze the stability of the 2 DOF suspension system, we have to make the model of the system. Here are symbols for modeling.

x, y: position of the suspended object

m: mass of the suspended object

 $x_m$ : position of the magnet about X axis

 $y_m$ : position of the magnet about Y axis

 $m_m$ : mass of the permanent magnet

 $f_x$ ,  $f_y$ : attractive force

 $f_{xx}$ ,  $f_{xy}$ ,  $f_{yx}$ ,  $f_{yy}$ : component of the force whose direction is indicated by the second subscript letter k: constant of the permanent magnet which defined as Eq. (1)

 $l_x$ ,  $l_y$ : length between the object and the respective axis magnet

 $k_p$ : proportional feedback gain

 $k_d$ : deferential feedback gain

 $x_{m0}, y_{m0}$ : equilibrium positions for the magnets

 $k_e$ : spring constant of the magnet suspension

 $k_c$ : dumping constant of the magnet suspension

 $F_x$ ,  $F_y$ : driving force for respective magnet, input of force control system

 $l_0$ : air gap length of equilibrium

g: gravity acceleration

#### Equation of movement of suspended object

We assume that the attractive forces act on the direction from the magnet tip to the center of the object, and are inversely proportional to the square of the air gap lengths. The attractive forces are represented by

$$f_x = \frac{k}{l_x^2} , \quad f_y = \frac{k}{l_y^2} \tag{1}$$

And the each components of the X and Y axes direction of the forces are represented by

$$f_{xx} = k(x_m - x)/l_x^3 \tag{2}$$

$$f_{xy} = -ky/l_x^3 \tag{3}$$

$$f_{yx} = -kx/l_y^3 \tag{4}$$

$$f_{yy} = k(y_m - y)/l_y^3 \tag{5}$$

where,

$$l_x = \sqrt{(x_m - x)^2 + y^2}$$
$$l_y = \sqrt{x^2 + (y_m - y)^2}$$

When the viscous friction of the air can be neglected, the equations of the movement of the suspended object about X and Y axis are

$$m\ddot{x} = f_{xx} + f_{yx} - mg/\sqrt{2} \tag{6}$$

$$m\ddot{y} = f_{xy} + f_{yy} - mg/\sqrt{2} \tag{7}$$

The outputs of the system are x and y position of the object and we assume that these position could be sensed by sensors.

#### Position Control Model and Force Control Model

We make two models of the suspension system. One is that the magnets are controlled by their positions. It means that the system inputs are the positions of the magnets. The other is the methods that the magnets are controlled by the actuator forces. In this case, the system inputs are the driving forces of the X and Y axes magnets.

We make the state space models for above two models respectively. This model is useful for multivariable system, and is usually a linear model. As the above equations have nonlinear terms, we have to linearize to make state space models.

In case of position control, system inputs are  $x_m$  and  $y_m$ , the magnet positions and system outputs are x and y, the positions of the object. The state variables are the positions and the velocities of the object. The state vector is represented by  $z_p$  and system input is  $u_p$ , and the state space equation is

$$\dot{z}_p = A_p z + B_p u_p \tag{8}$$

where.

$$z_p = (x \ y \ \dot{x} \ \dot{y})'$$

$$u_p = (\hat{x}_m \ \hat{y}_m)'$$

$$A_p = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ k_m/m & 0 & 0 & 0 \\ 0 & k_m/m & 0 & 0 \end{pmatrix}$$

$$B_p = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2k_m/m & 0 & 0 \\ 0 & -2k_m/m \end{pmatrix}$$

$$k_m = k/x_{m0}^3$$

and the hat represents the displacement from the equilibrium.

In case of force control, the system input  $u_f$  consists of  $F_x$  and  $F_y$  and the state vector  $z_f$  is made of the positions and the velocities of the object and two magnets. The equations of the movements of the permanent magnets are calculated as

$$m_m \ddot{x}_m = F_x - f_{xx} - \frac{\sqrt{2}}{2} m_m g - k_e x_m - k_c \dot{x}_m$$
 (9)

$$m_m \ddot{y}_m = F_y - f_{yy} - \frac{\sqrt{2}}{2} m_m g - k_e y_m - k_c \dot{y}_m$$
 (10)

The state space equation of force control model is

$$\dot{z}_f = A_f z_f + B_f u_f \tag{11}$$

where,

$$z_f = \begin{pmatrix} x & y & x_m & y_m & \dot{x} & \dot{y} & \dot{x}_m & \dot{y}_m \end{pmatrix}'$$

$$u_f = \begin{pmatrix} \hat{F}_x & \hat{F}_y \end{pmatrix}'$$

and

$$k_{mm} = -2k_m$$

$$k_{me} = (2k_m - k_e)/m_m$$

We can find that the above two system are controllable.

#### Controller

We use PD feedback control for the suspension system controller. Because the PD controller is very simple and it is good for the comparison of the systems.

When system inputs are the permanent magnet positions on X and Y axes, the magnet positions can be controlled by an operator without any delay. Open loop control can not make the system stable, feedback control is necessary. The controller senses the positions of the suspended object on both direction, and controls the position of each magnet based on the information of individual direction by PD control. The positions of magnets are calculated as

$$x_m = k_p x + k_d \dot{x} + x_{m0} \tag{12}$$

$$y_m = k_p y + k_d \dot{y} + y_{m0} \tag{13}$$

When inputs of the system are driving force and PD controller of the object is used, it is known that we have to support the magnets by elastic elements and dampers [5]. It means  $k_e \neq 0$ , and  $k_c \neq 0$ . The driving forces are calculated from the positions of the object and two magnets as

$$F_x = k_p x + k_d \dot{x} + F_{x0} \tag{14}$$

$$F_{\nu} = k_{\nu}y + k_{d}\dot{y} + F_{\nu 0} \tag{15}$$

Suspension component constants and feedback gains are same on X and Y axes.

#### **Numerical Simulation**

As the aim of the numerical simulation is having the knowledge of the stability of the 2 DOF suspension system considering nonlinear terms, the system constant can be normalized as

$$m = m_{mx} = m_{my} = k = k_c = x_{m0} = 1, ke = 6$$
 (16)

#### When Input Is Magnet Position

Simulations start at the following conditions. In the 1 DOF system, the initial position of the object is 0.5 near to the magnet from the equilibrium. It means the simulation starts 0.5 upper position. In the 2 DOF system, the initial position of the object is 0.5 near to the magnet along X axis from the equilibrium. It means the simulation starts at (x, y) = (0.5, 0).

In case of Stable Gain The feedback gains are set to  $(k_p, k_d) = (2, 0.3)$ . In the 1 DOF system, these gains make the system stable.

The response of the 1 DOF system is shown in the Fig. 4. The horizontal axis is time from simulation start, and the vertical axis represents the displacement of the suspended object. As shown in the figure, the response converges with vibration, and it is shown that the system is stable.

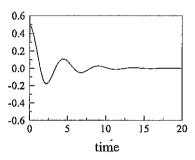


Fig. 4: Simulation Result of 1 DOF System  $(k_p, k_d) = (2, 0.3)$ 

The response of the 2 DOF system is shown in the Fig. 5 In the figure, we record the locus of the suspended object in the X-Y plane. The horizontal axis represents the displacement of the X direction, and the vertical axis is the Y direction. The arrow shows the direction of the movement. As shown in the figure, the object converges with drawing ovals. It is very similar to the result of the 1 DOF suspension system.

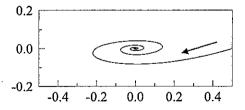


Fig. 5: Simulation Result of 2 DOF System  $(k_p, k_d)$ =(2, 0.3)

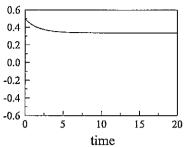


Fig. 6: Simulation Result of 1 DOF System  $(k_p, k_d)=(1, 0.3)$ 

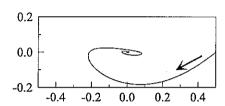


Fig. 7: Simulation Result of 2 DOF System  $(k_p, k_d)=(1, 0.3)$ .

In case of Critical Proportional Gain The simulation is carried out when the feedback gains are set to  $(k_p, k_d) = (1, 0.3)$ . In this case, the proportional gain  $k_p$  is on the border between stable and unstable in the 1 DOF system. This proportional gain is assumed to call a critical proportional gain.

In the 1 DOF system, the result of simulation is shown in Fig. 6. As the result is more clear, the initial velocity  $\dot{x} = -0.1$  is given. As shown in the figure, the object does not converges, nor diverges. It is floating state.

In the 2 DOF system, however, the result of simulation is shown in Fig. 7. An arrow in the figure indicates the movement direction of the object. As shown in the figure, we can see that the object stably converges to the origin. The reason is that the lateral force of the Y axis magnet increases the restoration force for the X axis movement of the object.

In case of Deferential Critical Gain. The feedback gains are set to  $(k_p, k_d) = (2, 0)$ , and simulation is carried out. This deferential gain  $k_d$  is the border gain of the stability in the 1 DOF system, and to call deferential critical gains.

In the 1 DOF system, the result of simulation is shown in Fig. 8. If the system is linear, the vibration continues with same amplitude. The system, however, is nonlinear, the vibration becomes gradually larger. Although it can not be seen in the figure.

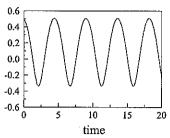


Fig. 8: Simulation Result of 1 DOF System  $(k_p, k_d) = (2, 0.)$ 

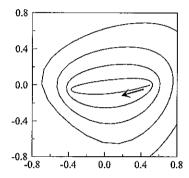


Fig. 9: Simulation Result of 2 DOF System  $(k_p, k_d)=(2, 0.)$ 

In the 2 DOF system, the result is shown in Fig. 9. As shown in the figure, the amplitude of vibration becomes rapidly large. This shows the dumping factor of the 2 DOF system is inferior to the 1 DOF system. The reason my be because the object moves in the lateral direction.

The above 3 example show that the 2 DOF system is different from the 1 DOF system, if the feedback gains are same. So the 2 DOF system can not be divided as individual subsystems of X and Y.

#### When Input Is Actuator Force

If the system inputs are the driving force of the permanent magnet, there are restrictions of the feedback gains[5]. In 1 DOF system, we can calculate the range of the stable gains easily. In 2 DOF system, however, the calculation is very complex. So we settle the deferential gain, and obtain the limitation of the proportional gain which makes the system stable.

We assume that the deferential gain  $k_d$  is 1.2, and calculate the limitation by Routh-Hurwitz criterion for stability. As the results, 1 DOF system is stable when the proportional gain is

$$6 < k_p < 6.32.$$
 (17)

and in 2 DOF system, the stable range is

$$4 < k_p < 5.12. (18)$$

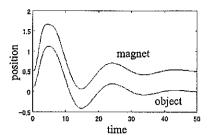


Fig. 10: Simulation Result of 1 DOF System with Force Input

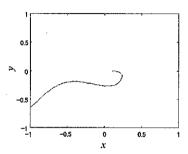


Fig. 11: Simulation Result of 2 DOF System with Force Input

In both system, there are upper and lower bounds. All values of 2 DOF system is smaller than that of 1 DOF system.

As the robustness is weak compared with the position input system, numerical simulations are carried out using the obtained feedback gains and small initial deviation. Here the feedback gains set to  $(k_p, k_d) = (6.1, 1.2)$  in 1 DOF system, and the  $(k_p, k_d) = (4.5, 1.2)$  in 2 DOF system. The initial position is (x, y) = (0.1, 0), and simulation starts.

The response of the 1 DOF system are shown in Fig. 10. It can be seen that the object converges the equilibrium. The response of the 2 DOF system are shown in Fig. 11. In the 2 DOF system, the system is not stable. As shown in the figure, the object falls down.

Next, the initial deviation is set to smaller than the above. The initial position is set to (x, y) = (0.03, 0). The response of these conditions is shown in Fig. 12. The gains are same as in Fig. 11. As shown in the figure, the system becomes stable suspension state. As the results, the initial deviation is important for the suspension system. The reason is that the system has nonlinear components for the attractive forces of magnets. This shows that the actual 2 DOF suspension system is delicate of the the equilibrium position adjusting.

# Conclusion

2 DOF magnetic suspension system which aims the micro manipulation function was proposed and

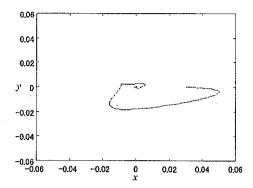


Fig. 12: Simulation Result of 2 DOF System with Force Input (initial deviation = 0.03)

a prototype system was introduced. The feature of the suspension system is that it consists of permanent magnets and linear actuators. The proposed 2 DOF system was modeled, and numerical simulations were carried out comparing with a 1 DOF system. The feedback gains which make the system stable is different between the 2 DOF system and the 1 DOF system.

As further study, the robust control system theory is introduced to the system and the initial deviation for the stable state will be expanded. The examination of the prototype system is carried out and feasibility of the 2 DOF manipulation is verified.

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